

Wireless Communication Systems

BEC703

CREDITS: 4

Course Objectives:

- Understand the concepts of signal propagation over wireless channels
- Understand the multiple access techniques used in cellular communications standards.
- Understand the system architecture and layers of LTE based on the use of OFDMA and SCFDMA principles.
- Understand the design and coding of MIMO wireless systems .

Module 1 - Principles of Wireless Communications

1. The Wireless Communication Environment
2. Modeling of wireless systems
3. System model for narrowband Signals
4. Rayleigh fading Wireless Channel.

Module 1 - The Wireless Channel

1. Basics of Wireless Channel Modelling
2. Average Delay Spread in Outdoor Cellular Channels
3. Coherence Bandwidth in Wireless Communications
4. Relation between ISI and Coherence Bandwidth,
5. Doppler Fading in Wireless Systems
6. Doppler Impact on a Wireless Channel
7. Coherence Time of the Wireless Channel

Module 2 - Code Division for Multiple Access (CDMA)

1. Introduction to CDMA
2. Basic CDMA Mechanism
3. Fundamentals of CDMA Codes
4. Spreading Codes based on Pseudo-Noise (PN) Sequences
Properties of PN Sequences
5. Correlation Properties of Random CDMA Spreading Sequences
6. Advantages of CDMA

Module 2 -Orthogonal Frequency Division Multiplexing (OFDM)

1. Introduction
2. Motivation and Multicarrier basics
3. OFDM basics
4. OFDM Example
5. MIMO OFDM
6. OFDM Peak to Average Power Ratio
7. SC-FDMA

Module 3 - Evolution of Cellular Technologies

1. 1G – First Generation Cellular Systems
2. 2G – Digital Cellular Systems (GSM) and its Evolution
3. 3G – Broadband Wireless Systems
4. LTE – Key Enabling Technologies & Features
5. LTE Network Architecture

Module 3 - Frequency Domain Multiple Access

1. Multiple Access for OFDM Systems
2. Orthogonal Frequency Division Multiple Access (OFDMA)
3. Single Carrier Frequency Division Multiple Access (SC-FDMA)

Module 4 - Multiple Input Multiple Output (MIMO) Wireless Communications

1. Introduction to MIMO Communications
2. MIMO System Model
3. MIMO Zero Forcing Receiver
4. MIMO MMSE Receiver
5. Singular Value Decomposition (SVD) of MIMO Channel
6. SVD and MIMO Capacity
7. Alamouti and Space-Time Codes
8. Nonlinear MIMO Receiver: V-BLAST
9. MIMO Beamforming

Module 5

Overview and Channel Structure of LTE:

1. - Radio Interface Architecture
2. - LTE Design Principles
3. - Network Architecture
4. - Radio Interface Protocols

Hierarchical Structure of LTE:

- 1.** - Logical Channels
2. - Transport Channels
3. - Physical Channels
4. - Channel Mapping

Radio Resources:

1. - Downlink OFDMA Radio Resources
2. - Physical Resource Blocks (PRB) for OFDMA
3. - Uplink SC-FDMA Radio Resources

Course Outcomes

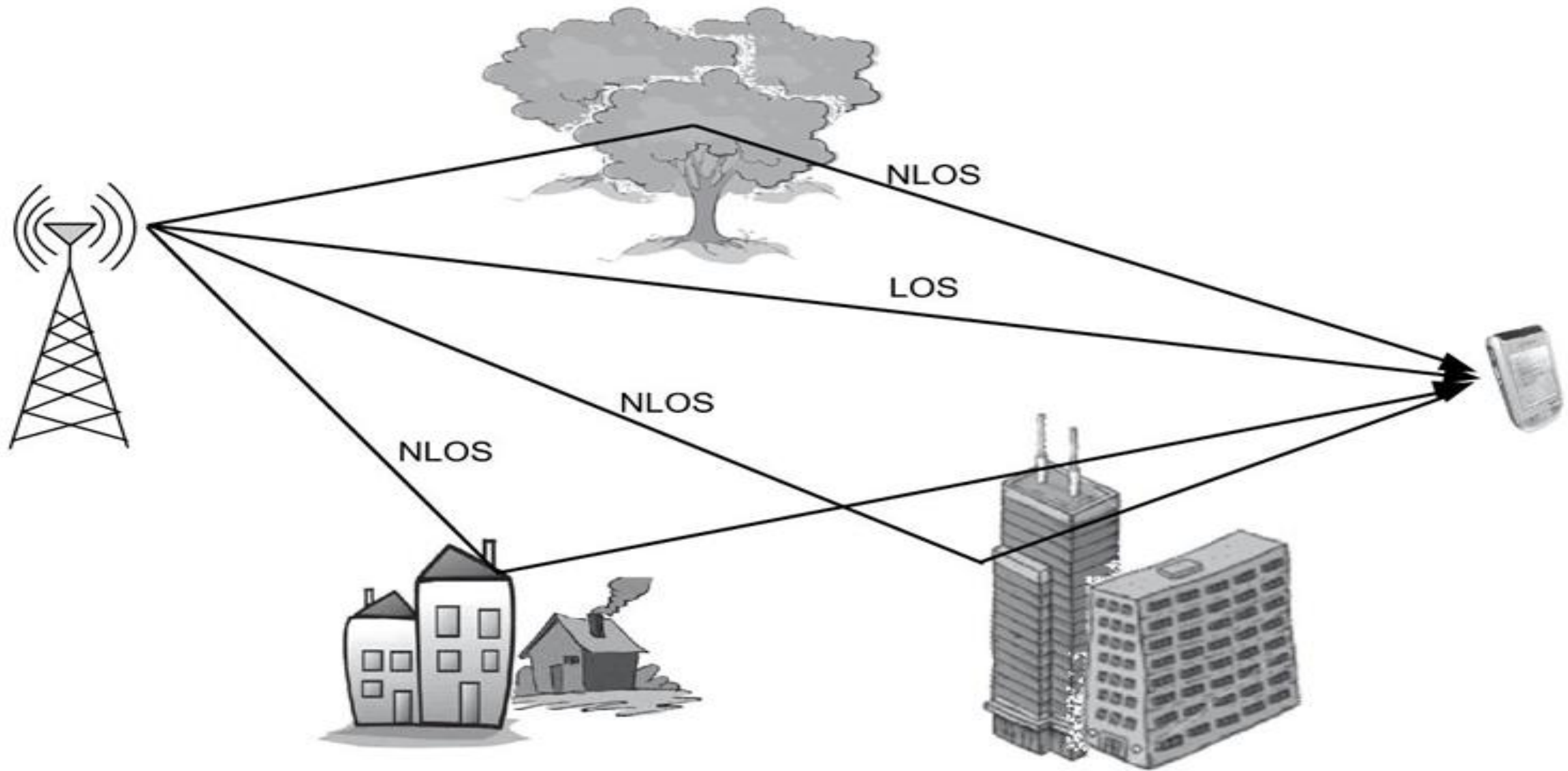
1. Describe the wireless channel models for slow and fast fading environments.
2. Understand the different multiple access technologies used in wireless communications.
3. Understand the system architecture and the functional standard specified in LTE 4G.
4. Describe the MIMO transmitter and receiver process using coding examples.

Module 1 - Principles of Wireless Communications

1. The Wireless Communication Environment
2. Modeling of wireless systems
3. System model for narrowband Signals
4. Rayleigh fading Wireless Channel.

1. The Wireless Communication Environment

- **Wireline systems:** Single fixed path (e.g., coaxial cable, twisted pair).
- **Wireless systems:** Multiple paths (direct, reflected, scattered).
- **Multipath interference:** Superposition of multiple signal copies.
- **Each path has different attenuations**, delays, and phase shifts.
- This causes fading (variation in received signal strength).
- **Deep fade:** Severe drop in SNR → Temporary communication failure.
- **Key difference:** Fading in wireless requires new system design approaches



Schematic of the wireless-propagation environment

2. Modelling of Wireless Systems

Purpose: Understand wireless environment and quantify system performance.

Need: Accurate analytical models to represent wireless channels.

Approach: Use baseband and passband signal representation.

Passband Signal Representation

$$s(t) = \text{Re} \left\{ s_b(t) e^{j2\pi f_c t} \right\}$$

$s_b(t)$ → complex baseband representation of the transmitted signal.

f_c → carrier frequency.

2. Modelling of Wireless Systems

Time-Invariant Wireless Channel Model

Assume channel is Linear Time-Invariant(LTI) initially.

Channel has **L** multipath components.

-Each path has:

1. Attenuation factor (a_i)
2. Path delay (τ_i)

Impulse response of single path

$$h_i(\tau) = a_i \delta(\tau - \tau_i)$$

2. Modelling of Wireless Systems

Time-Invariant Wireless Channel Model

Multipath Channel Impulse Response (CIR)

$$h(\tau) = \sum_{i=0}^{L-1} a_i \delta(\tau - \tau_i)$$

- Known as “Tapped Delay Line Model”.
- Paths can be NLOS (reflections/scattering) or LOS.

2. Modelling of Wireless Systems

Received Signal in Passband

$$y(t) = s(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) s(t - \tau) d\tau$$

By inserting the expression for the tapped delay-line channel

$$y(t) = \sum_{i=0}^{L-1} a_i \int_{-\infty}^{\infty} \delta(\tau - \tau_i) s(t - \tau) d\tau = \sum_{i=0}^{L-1} a_i s(t - \tau_i)$$

2. Modelling of Wireless Systems

The received signal can be written in terms of the transmitted base band signal

$$y(t) = \operatorname{Re} \left\{ \sum_{i=0}^{L-1} a_i s_b(t - \tau_i) e^{j2\pi f_c(t - \tau_i)} \right\}$$
$$= \operatorname{Re} \left\{ \underbrace{\left(\sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} s_b(t - \tau_i) \right)}_{y_b(t)} e^{j2\pi f_c t} \right\}$$

2. Modelling of Wireless Systems

$y_b(t)$, the complex base band signal equivalent of the received signal $y(t)$.

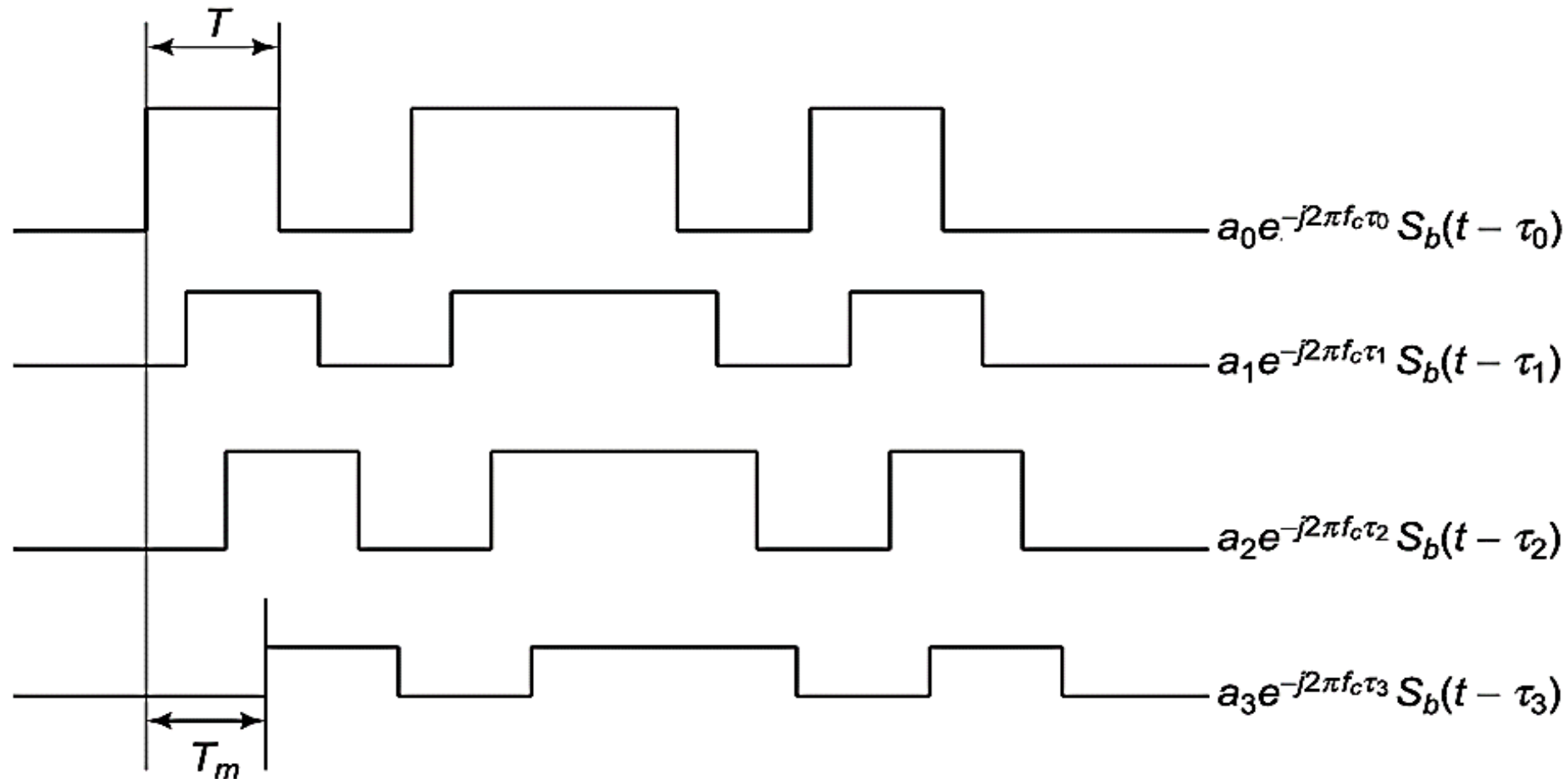
$$y_b(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} s_b(t - \tau_i)$$

The received baseband signal consists of multiple delayed copies

1. The attenuation factor a_i
2. The path delay τ_i
3. The phase factor $e^{-j2\pi f_c \tau_i}$

2. Modelling of Wireless Systems

Multipath signal components at the receiver



2. Modelling of Wireless Systems

Consider a wireless signal with a carrier frequency of $f_c = 850$ MHz, which is transmitted over a wireless channel that results in $L = 4$ multipath components at delays of 201, 513, 819, 1223 ns and corresponding to received signal amplitudes of 1, 0.6, 0.3, 0.2 respectively. Derive the expression for the received baseband signal $y_b(t)$ if the transmitted baseband signal is $s_b(t)$.

2. Modelling of Wireless Systems

Given

- Carrier frequency: $f_c = 850$ MHz
- Number of paths: $L = 4$
- Delays: $\tau_0 = 201$ ns, $\tau_1 = 513$ ns, $\tau_2 = 819$ ns, $\tau_3 = 1223$ ns
- Amplitudes: $a_0 = 1$, $a_1 = 0.6$, $a_2 = 0.3$, $a_3 = 0.2$
- Transmitted baseband signal: $s_b(t)$

Baseband received-signal formula

$$y_b(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} s_b(t - \tau_i)$$

$$\begin{aligned} a_0 e^{-j2\pi f_c \tau_0} &= 1 \times e^{-j2\pi 850 \times 10^6 \times 201 \times 10^{-9}} \\ &= 0.59 + j0.81 \end{aligned}$$

2. Modelling of Wireless Systems

The receive base band signal is

$$y_b(t) = (0.59 + j0.81) s_b(t) + (0.57 - j0.19) s_b(t - 201 \times 10^{-9}) \\ + (0.18 - j0.24) s_b(t - 513 \times 10^{-9}) + (-0.19 + j0.06) s_b(t - 1223 \times 10^{-9})$$

3. System Model for Narrowband Signals

Narrowband signal approximation,

$$y_b(t) = \underbrace{\left(\sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} \right)}_{ae^{j\phi}} s_b(t) = h s_b(t)$$

$$h = ae^{j\phi} = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

Complex fading channel coefficient

4. Rayleigh Fading Wireless Channel

Complex fading channel coefficient

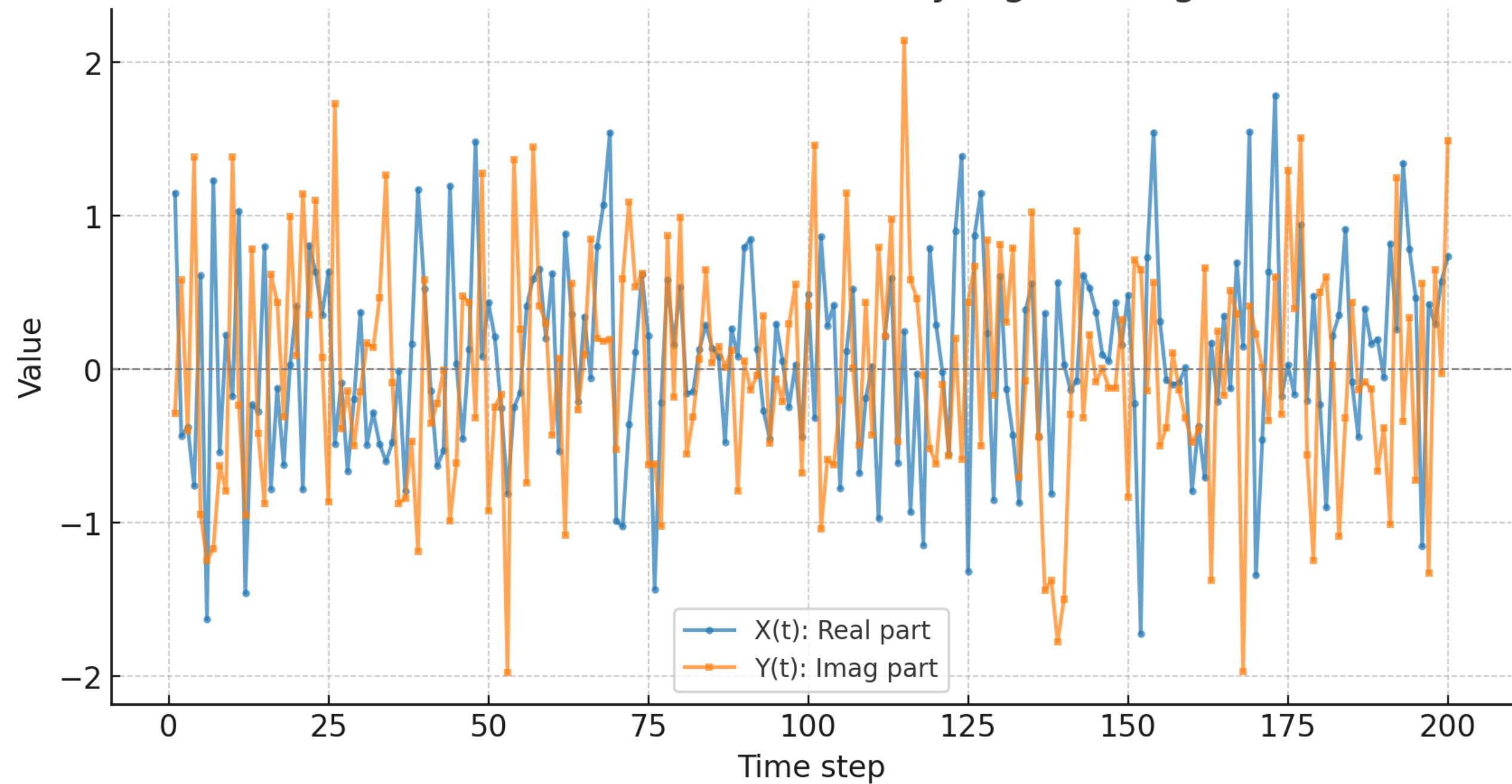
$$h = ae^{j\phi} = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

Expressed in terms of its real and imaginary components as

$$h = ae^{j\phi} = \sum_{i=0}^{L-1} (x_i + jy_i) = X + jY$$

Assume X,Y are random in nature.

Random Nature of X and Y in a Rayleigh Fading Channel

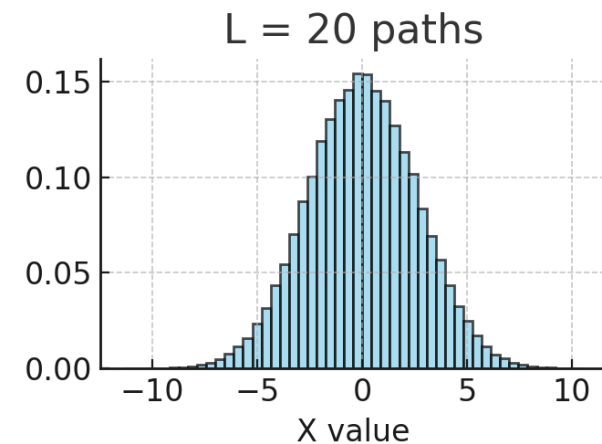
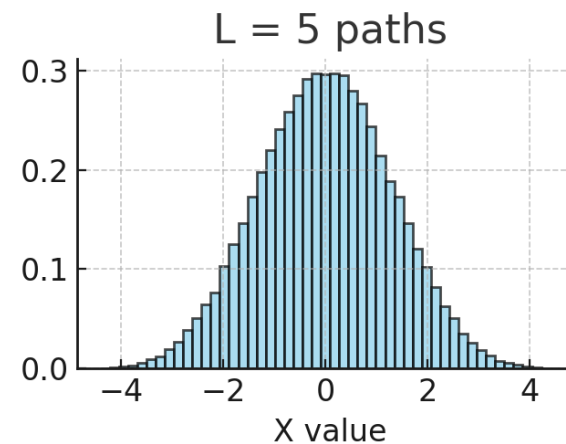
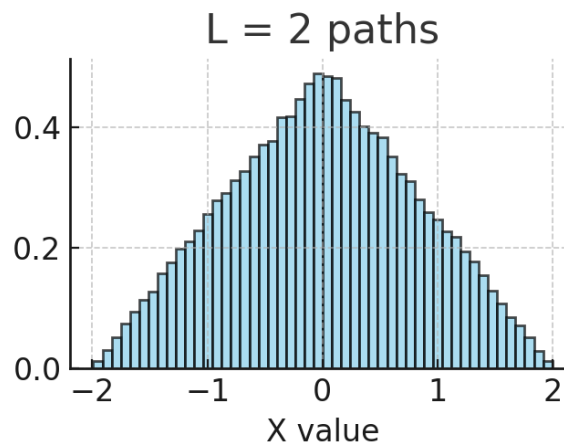
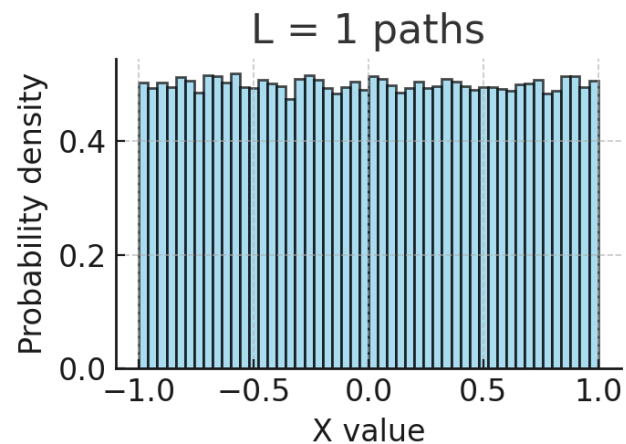


4. Rayleigh Fading Wireless Channel

- Assume X, Y are **Gaussian** and un-correlated

*Central Limit Theorem (CLT): states that a **normalized random variable** derived from the **sum of a large number of independent and identically distributed random components** will **converge to a Gaussian (normal) random variable**.*

Central Limit Theorem: Sum of L Non-Gaussian Path Components Approaches Gaussian



4. Rayleigh Fading Wireless Channel

1D Standard Gaussian:

If $X \sim N(\mu, \sigma^2)$, the PDF is:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

4. Rayleigh Fading Wireless Channel

- Assumption holds as $L \rightarrow \infty$ (large number of multipath components).
- $X, Y \sim N\left(0, \frac{1}{2}\right) \rightarrow$ Zero mean, variance $\frac{1}{2}$.
- X and Y are Gaussian and uncorrelated $\Rightarrow$ **Independent**.
- Joint PDF given by standard bivariate Gaussian:

$$f_{X,Y}(x, y) = \frac{1}{\pi} e^{-(x^2 + y^2)}$$

4. Rayleigh Fading Wireless Channel

Joint PDF of Amplitude and Phase

Given:

- $X, Y \sim N\left(0, \frac{1}{2}\right)$ — independent Gaussian variables
- $Z = X + jY$ — complex Gaussian

When you change variables from (X, Y) to (A, Φ) , you need the **Jacobian determinant**:

Transform to Polar Coordinates:

- $A = \sqrt{X^2 + Y^2}$ — **Amplitude**
- $\Phi = \tan^{-1}(Y/X)$ — **Phase**

$$J_{X,Y} = \left| \begin{bmatrix} \cos \phi & \sin \phi \\ -a \sin \phi & a \cos \phi \end{bmatrix} \right| = a$$

4. Rayleigh Fading Wireless Channel

Joint PDF of Amplitude and Phase

$$f_{X,Y}(x, y) = \frac{1}{\pi} e^{-(x^2 + y^2)} \quad x = a \cos \phi, \quad y = a \sin \phi$$

$$f_{A,\Phi}(a, \phi) = \frac{1}{\pi} e^{-a^2} J_{X,Y}$$

$$f_{A,\Phi}(a, \phi) = \frac{a}{\pi} e^{-a^2}$$

Given Joint PDF

$$f_{A,\Phi}(a, \phi) = \frac{1}{\pi} e^{-a^2} \cdot a$$

where $a \geq 0$ (amplitude) and $-\pi < \phi \leq \pi$ (phase).

To find the marginal PDF of amplitude $f_A(a)$, integrate over all possible values of ϕ :

$$f_A(a) = \int_{-\pi}^{\pi} f_{A,\Phi}(a, \phi) d\phi$$

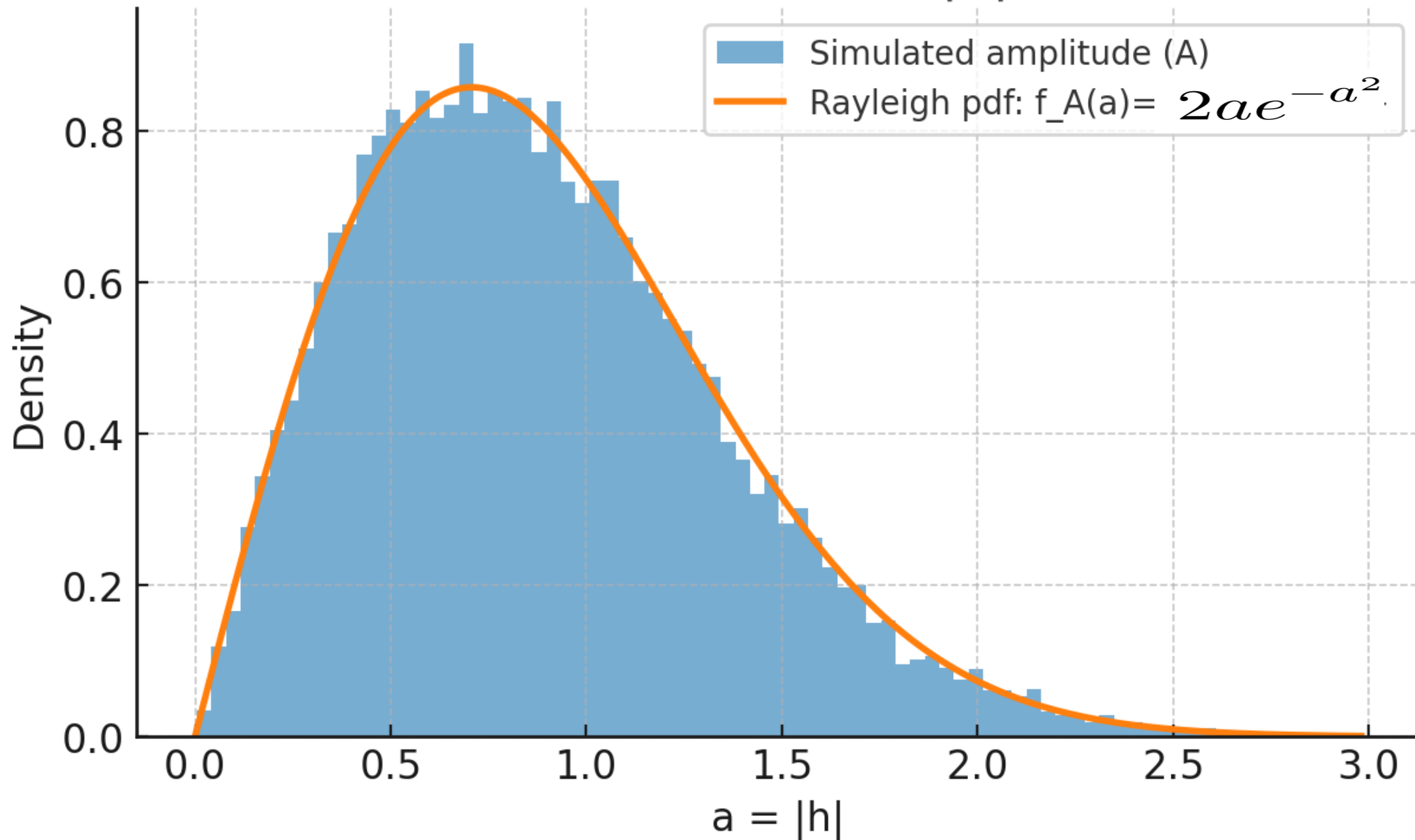
Since the joint PDF does not depend on ϕ , the integration becomes:

$$f_A(a) = \frac{1}{\pi} e^{-a^2} \cdot a \cdot (2\pi)$$

$$f_A(a) = 2a e^{-a^2}, \quad 0 \leq a < \infty$$

This is the **Rayleigh distribution**, which commonly describes the amplitude of a multipath fading signal.

Amplitude Distribution of $|h|$ (Rayleigh)



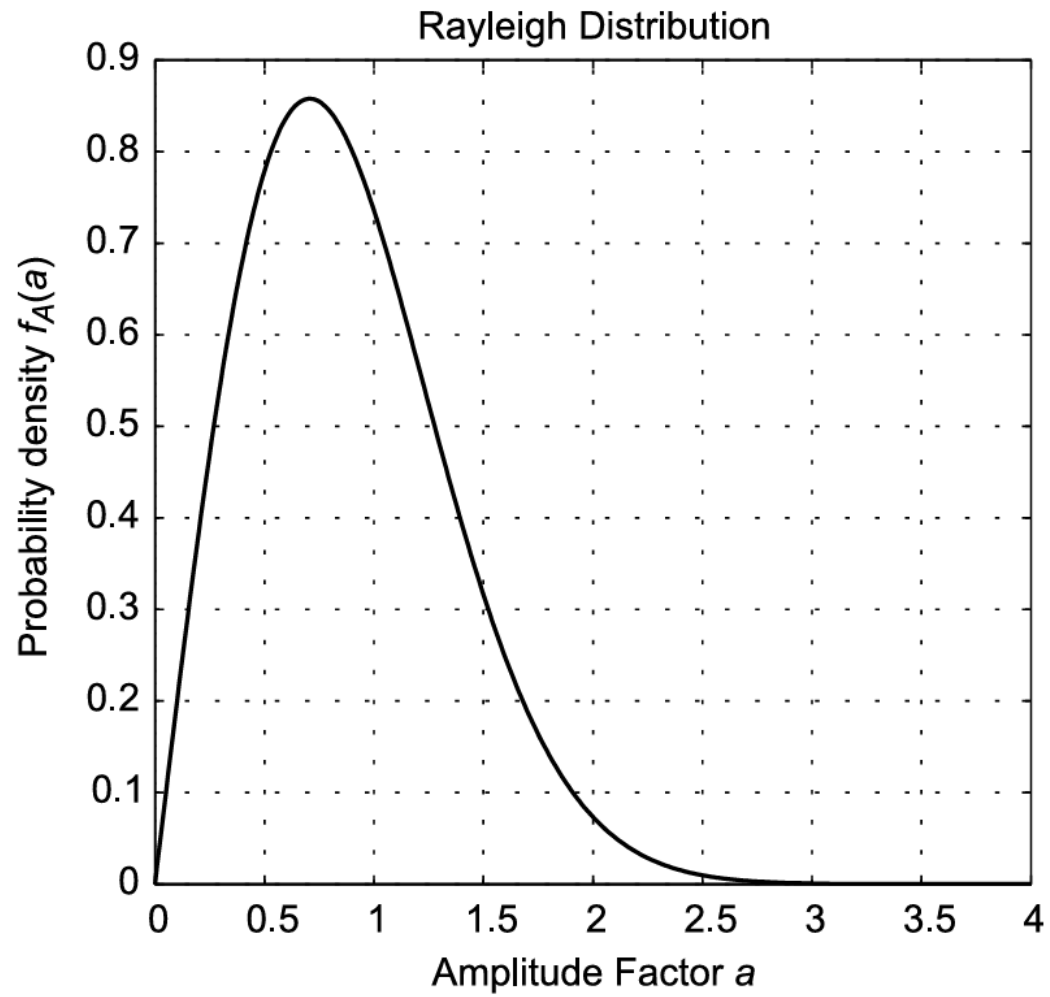


Figure 3.3 *Rayleigh density for amplitude factor a*

For the **marginal PDF of phase** $f_{\Phi}(\phi)$, integrate over all possible values of a :

$$f_{\Phi}(\phi) = \int_0^{\infty} f_{A,\Phi}(a, \phi) da$$

Substituting the joint PDF:

$$f_{\Phi}(\phi) = \frac{1}{\pi} \int_0^{\infty} a e^{-a^2} da$$

Using the substitution $u = a^2$, $du = 2a da$, we get:

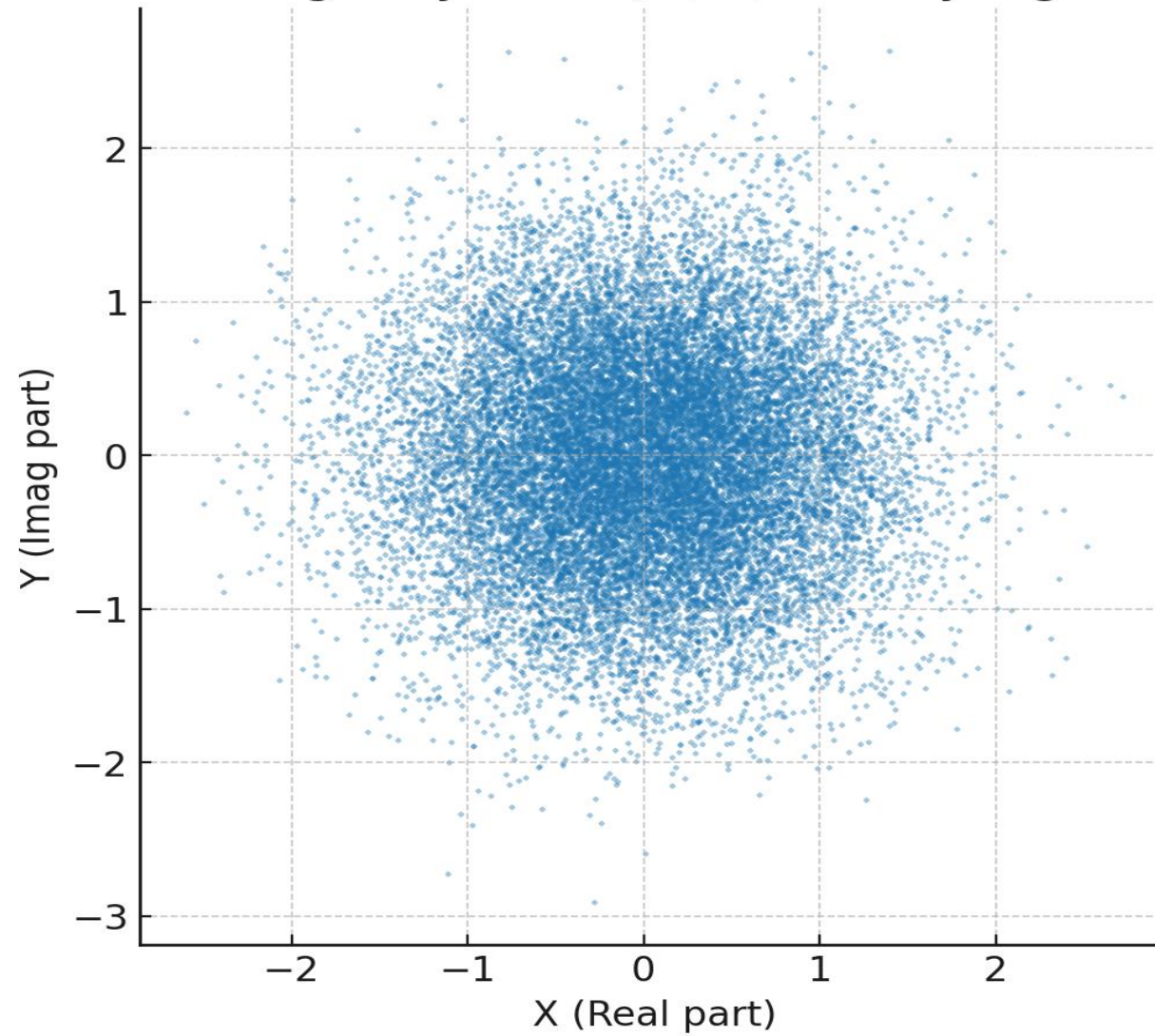
$$f_{\Phi}(\phi) = \frac{1}{\pi} \cdot \frac{1}{2} \int_0^{\infty} e^{-u} du$$

The integral equals 1, so:

$$f_{\Phi}(\phi) = \frac{1}{2\pi}$$

This is a **uniform distribution**, meaning the phase is equally likely anywhere between $-\pi$ and π .

Scatter of Real vs Imaginary Parts (X, Y) for Rayleigh Fading Coefficient



4. Rayleigh Fading Wireless Channel

$$f_{A,\Phi}(a, \phi) = f_A(a) f_{\Phi}(\phi)$$

- The amplitude A follows a Rayleigh distribution, modeling the random fluctuations due to multipath fading.
- The phase Φ follows a uniform distribution, representing the completely random phase shift caused by the channel.
- Average power in the amplitude a of the Rayleigh fading channel coefficient

$$\mathbb{E} \left\{ |h|^2 \right\} = \mathbb{E} \left\{ a^2 \right\} = \mathbb{E} \left\{ X^2 + Y^2 \right\} = 1$$

4. Baseband Model of a Wireless System

System Model:

$$y(k) = h x(k) + n(k)$$

- $x(k)$: Transmitted symbol (BPSK, QPSK, etc.)
- h : Rayleigh fading coefficient (random, time-varying)
- $n(k)$: AWGN noise (unit variance)
- $y(k)$: Received signal

Special Case: Wireline System

- No fading $\rightarrow h = 1$
- Model reduces to:

$$y(k) = x(k) + n(k)$$

Key Insights

- Wireless channels $\rightarrow$ multipath fading (random amplitude & phase)
- Wireline channels $\rightarrow$ only AWGN, no fading

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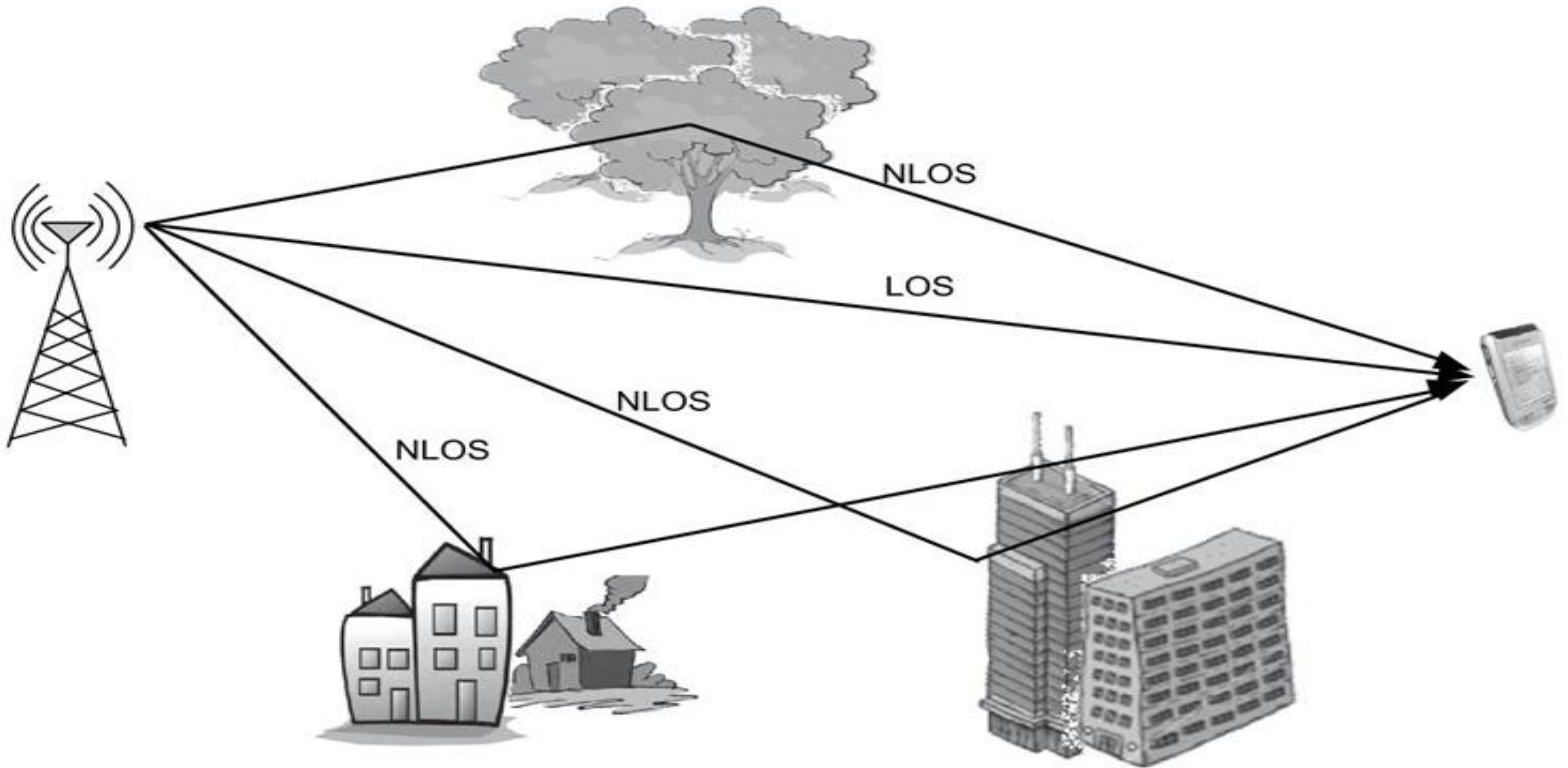
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Schematic of the wireless-propagation environment

1 -Basics of Wireless Channel Modelling

Multipath Nature of Wireless Channels

- In real wireless environments, the transmitted signal doesn't travel by just one straight line.
- Instead, it **bounces and scatters** off objects like **buildings, trees, vehicles, and water bodies**.
- This produces **multiple copies (multipath components)** of the same signal that arrive at the receiver:
 - Each with **different delay** (τ_i)
 - Each with **different attenuation/amplitude** (a_i)
- This is why wireless signals fade, distort, or interfere with themselves.

1 -Basics of Wireless Channel Modelling

Impulse Response of Multipath Channel

The channel is described mathematically as:

$$h(t) = \sum_{i=0}^{L-1} a_i \delta(t - \tau_i)$$

where:

- L = number of multipath components
- a_i = attenuation (complex value) of the i-th path
- τ_i = time delay of the i-th path
- $\delta(t - \tau_i)$ = impulse shifted by delay τ_i

1 -Basics of Wireless Channel Modelling

Multipath Power Profile

We often care about the **power** in each path, not just amplitude.

- Power gain of each path:

$$g_i = |a_i|^2$$

- Power profile:

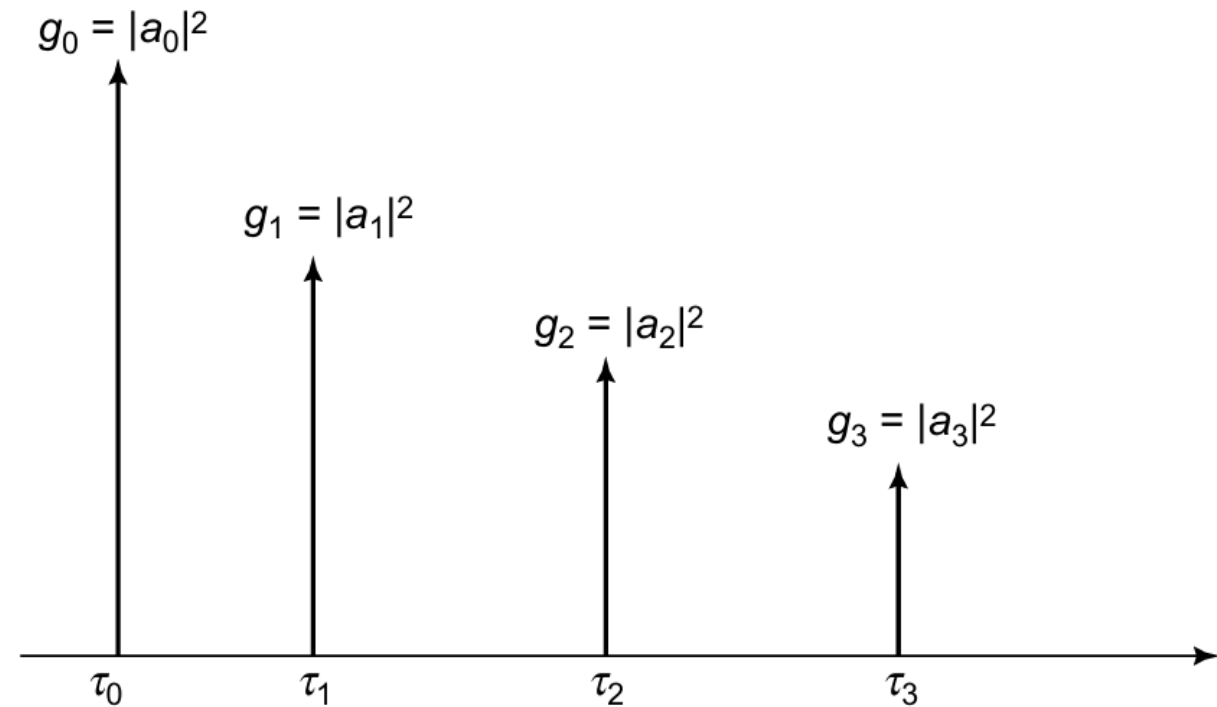
$$\phi(t) = \sum_{i=0}^{L-1} g_i \delta(t - \tau_i)$$

This tells us how much signal power arrives at what delay.

1 -Basics of Wireless Channel Modelling

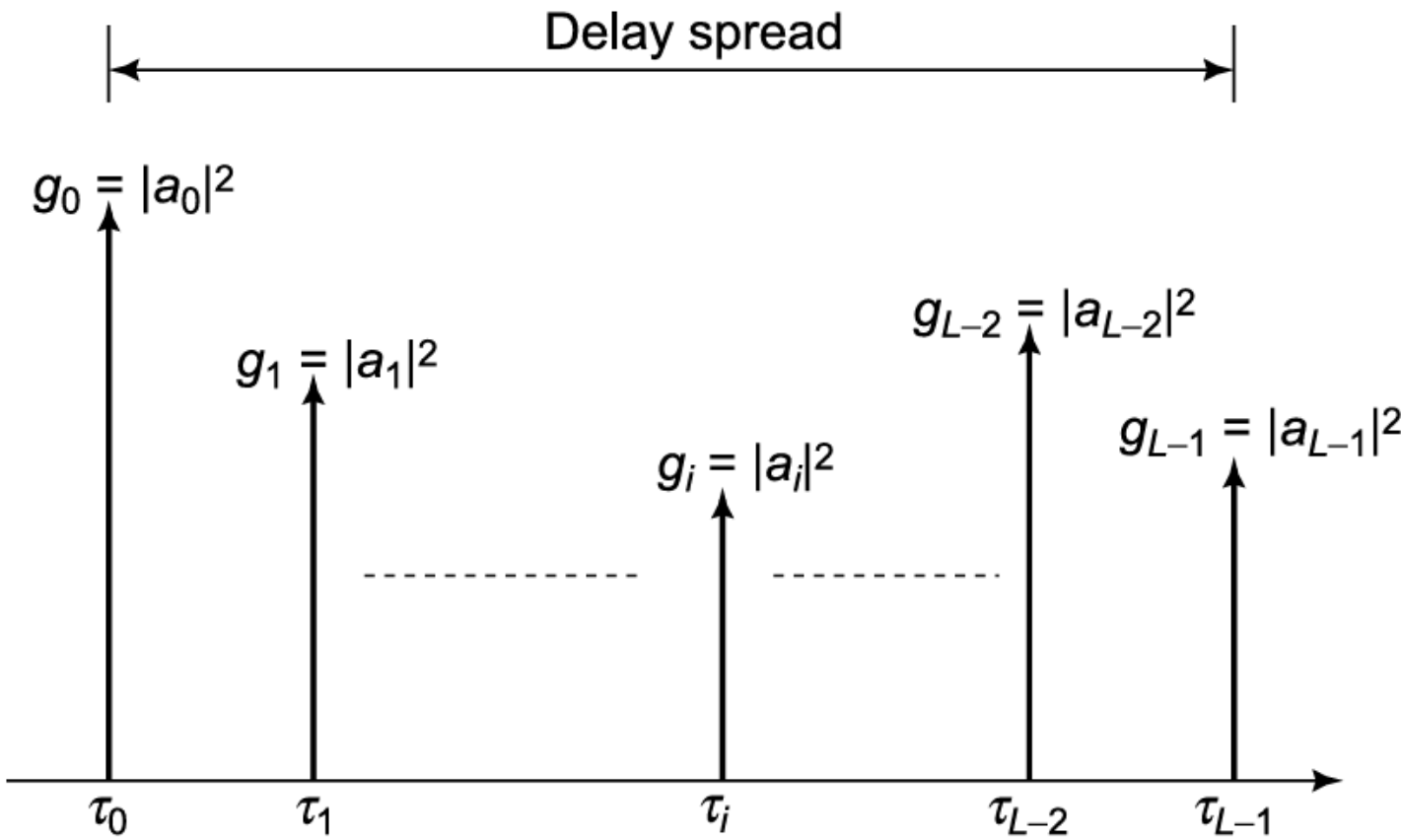
Example: L = 4 Multipath Channel

Gain	Delay
$ a_0 ^2$	τ_0
$ a_1 ^2$	τ_1
$ a_2 ^2$	τ_2
$ a_3 ^2$	τ_3



Schematic of an L = 4 tap wireless channel profile

1 -Basics of Wireless Channel Modelling



Schematic of a typical wireless channel power profile and delay spread

1 -Basics of Wireless Channel Modelling

Delay Spread

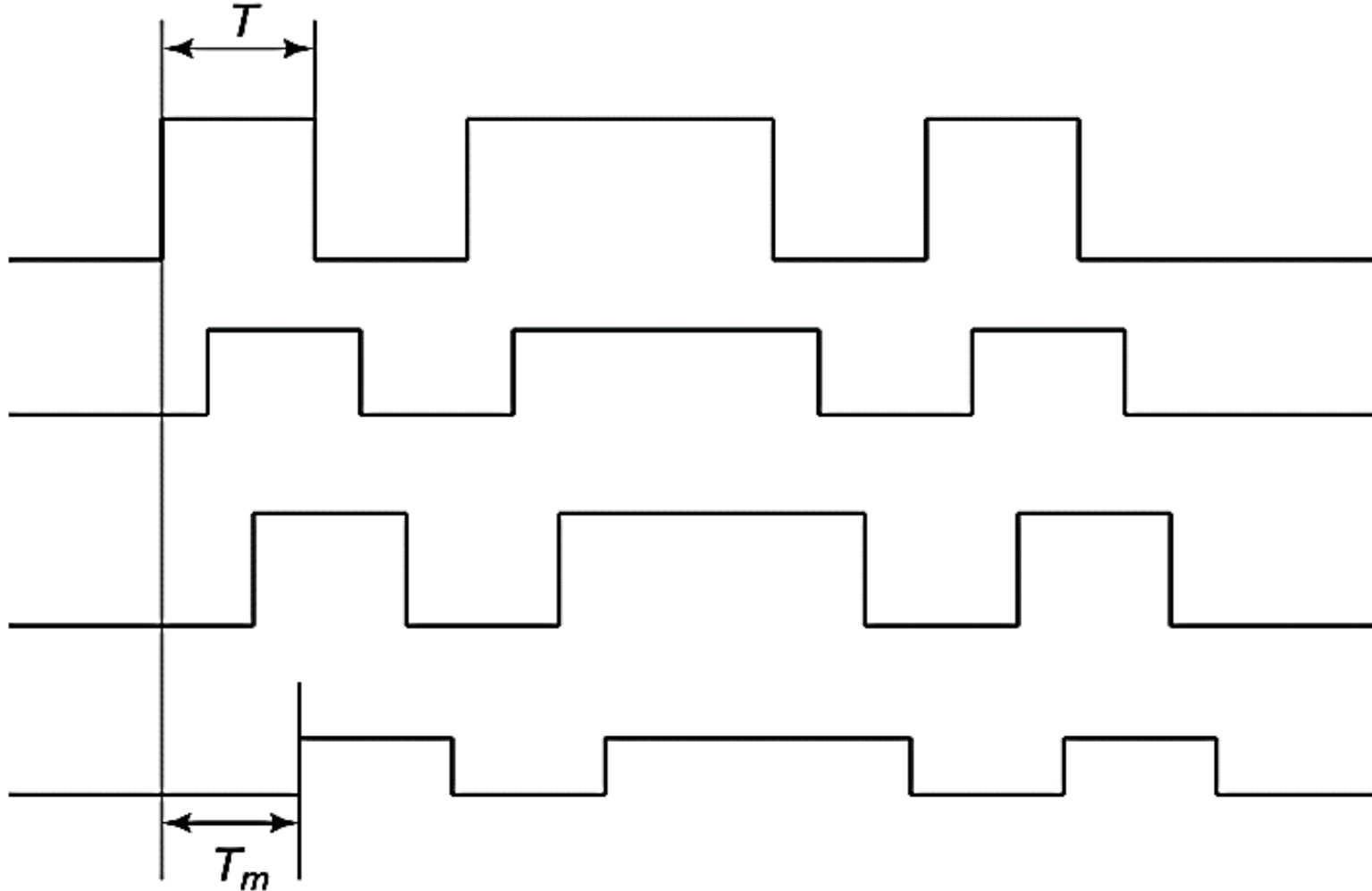
- Since parts of the signal arrive at **different times**, the received power is spread out.
- This spreading is called the **delay spread** of the channel.
- Delay spread = measure of **time dispersion** in multipath environment.
- It is a **key parameter** because:
 - Larger delay spread → more **inter-symbol interference (ISI)**
 - Smaller delay spread → channel looks “cleaner,” like wireline.

1 -Basics of Wireless Channel Modelling

Delay Spread

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1 -Basics of Wireless Channel Modelling



1 -Basics of Wireless Channel Modelling

Maximum Delay Spread $\sigma_{\tau}^{\max}$

The **maximum delay spread** of a wireless channel is the difference between the arrival time of the **last multipath component** and the **first multipath component**

$$\sigma_{\tau}^{\max} = \tau_{L-1} - \tau_0$$

Where:

- $\tau_0 \rightarrow$ delay of the **first arriving path**
- $\tau_{L-1} \rightarrow$ delay of the **last arriving path**
- $L \rightarrow$ number of multipath components

1 -Basics of Wireless Channel Modelling

Maximum Delay Spread $\sigma_{\tau}^{\max}$

- It measures the **time interval over which the multipath energy is spread**.
- indicates:
 - Rich scattering environment
 - Stronger multipath effects
 - Larger differential propagation delays between signal copies
- The value depends only on the **difference** between the first and last path delays, not their absolute values.

1 -Basics of Wireless Channel Modelling

Maximum Delay Spread $\sigma_{\tau}^{\max}$

Important Property

- The **distance** of the mobile from the base station does **not** affect delay spread.
- Example:
 - If there is only **one propagation path** with delay τ_0 , then:

$$\sigma_{\tau}^{\max} = \tau_0 - \tau_0 = 0$$

- Even if the mobile is far away (large τ_0), the delay spread is **zero** because there is no multipath.
- Thus, delay spread depends on **multipath richness**, not absolute propagation time.

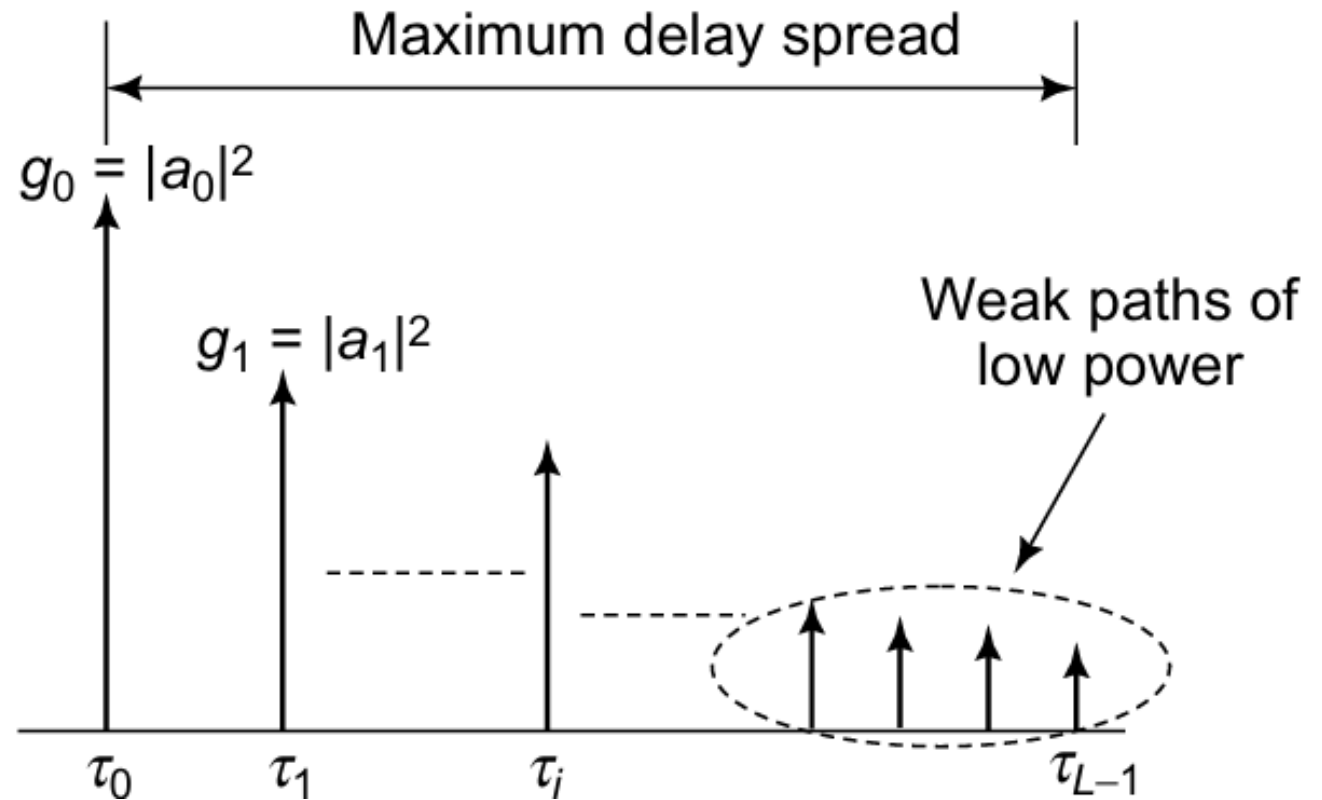
1 -Basics of Wireless Channel Modelling

Maximum Delay Spread $\sigma_{\tau}^{\max}$

Consider an $L = 4$ multipath channel with the delays τ_0, τ_{L-1} corresponding to the first and last arriving paths, given as $\tau_0 = 0 \mu s$ and $\tau_{L-1} = 5 \mu s$. Such a wireless-channel power profile is shown schematically in Figure 4.4. What is the maximum delay spread $\sigma_{\tau}^{\max}$ corresponding to this wireless channel?

1 -Basics of Wireless Channel Modelling

RMS Delay Spread $\sigma_{\tau}^{\text{RMS}}$



1 -Basics of Wireless Channel Modelling

RMS Delay Spread $\sigma_{\tau}^{\text{RMS}}$

- Multipath propagation: signals arrive via multiple paths (early strong paths, late weak paths).
- **Maximum Delay Spread** = time difference between first and last arriving path.
- Limitation: Late weak paths may exaggerate delay spread and give misleading results.
- **RMS Delay Spread (σ_{rms})** = measures actual time dispersion.
- Weights each path according to its received power.
- Provides realistic indicator of multipath effects.
- Larger σ_{rms} $\rightarrow$ higher Inter Symbol Interference (ISI).

1 -Basics of Wireless Channel Modelling

RMS Delay Spread $\sigma_{\tau}^{\text{RMS}}$

Discrete multipath power profile

$$g_i = |a_i|^2, \quad 0 \leq i \leq L - 1$$

Define the power fraction

$$b_i \triangleq \frac{g_i}{g_0 + g_1 + \cdots + g_{L-1}}, \quad b_i > 0, \quad \sum_{i=0}^{L-1} b_i = 1$$

1 -Basics of Wireless Channel Modelling

RMS Delay Spread $\sigma_{\tau}^{\text{RMS}}$

Average delay

$$\bar{\tau} = b_0\tau_0 + b_1\tau_1 + \dots + b_{L-1}\tau_{L-1} = \sum_{i=0}^{L-1} b_i\tau_i$$

$$= \sum_{i=0}^{L-1} \frac{g_i}{\sum_{j=0}^{L-1} g_j} \tau_i$$

$$= \frac{\sum_{i=0}^{L-1} g_i\tau_i}{\sum_{j=0}^{L-1} g_j}$$

1 -Basics of Wireless Channel Modelling

RMS Delay Spread $\sigma_{\tau}^{\text{RMS}}$

$$(\sigma_{\tau}^{\text{RMS}})^2 = b_0 (\tau_0 - \bar{\tau})^2 + b_1 (\tau_1 - \bar{\tau})^2 + \dots + b_{L-1} (\tau_{L-1} - \bar{\tau})^2$$

$$= \sum_{i=0}^{L-1} b_i (\tau_i - \bar{\tau})^2$$

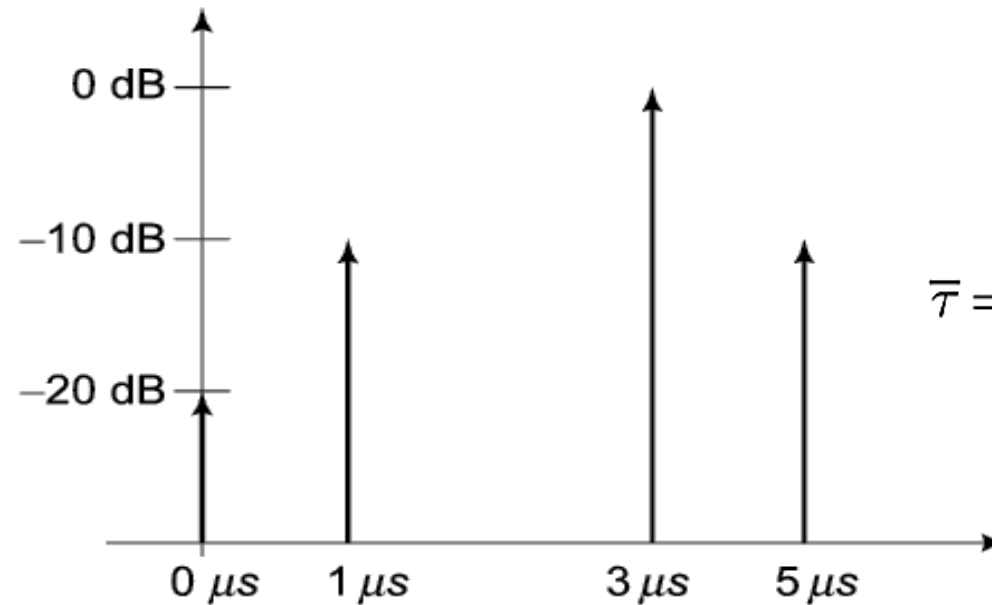
$$\sigma_{\tau}^{\text{RMS}} = \sqrt{\frac{\sum_{i=0}^{L-1} g_i (\tau_i - \bar{\tau})^2}{\sum_{i=0}^{L-1} g_i}}$$

$$= \sqrt{\frac{\sum_{i=0}^{L-1} |a_i|^2 (\tau_i - \bar{\tau})^2}{\sum_{i=0}^{L-1} |a_i|^2}}$$

1 -Basics of Wireless Channel Modelling

RMS Delay Spread $\sigma_{\tau}^{\text{RMS}}$

Consider the multipath power profile of a wireless channel shown in Figure 4.6 comprising $L = 4$ multipath components. Compute the RMS delay spread $\sigma_{\tau}^{\text{RMS}}$ for this wireless channel.



$$\sigma_{\tau}^{\text{RMS}} = \sqrt{\frac{\sum_{i=0}^{L-1} g_i (\tau_i - \bar{\tau})^2}{\sum_{i=0}^{L-1} g_i}}$$

$$\bar{\tau} = b_0\tau_0 + b_1\tau_1 + \dots + b_{L-1}\tau_{L-1} = \sum_{i=0}^{L-1} b_i\tau_i$$

$$b_i \triangleq \frac{g_i}{g_0 + g_1 + \dots + g_{L-1}}$$

1 -Basics of Wireless Channel Modelling

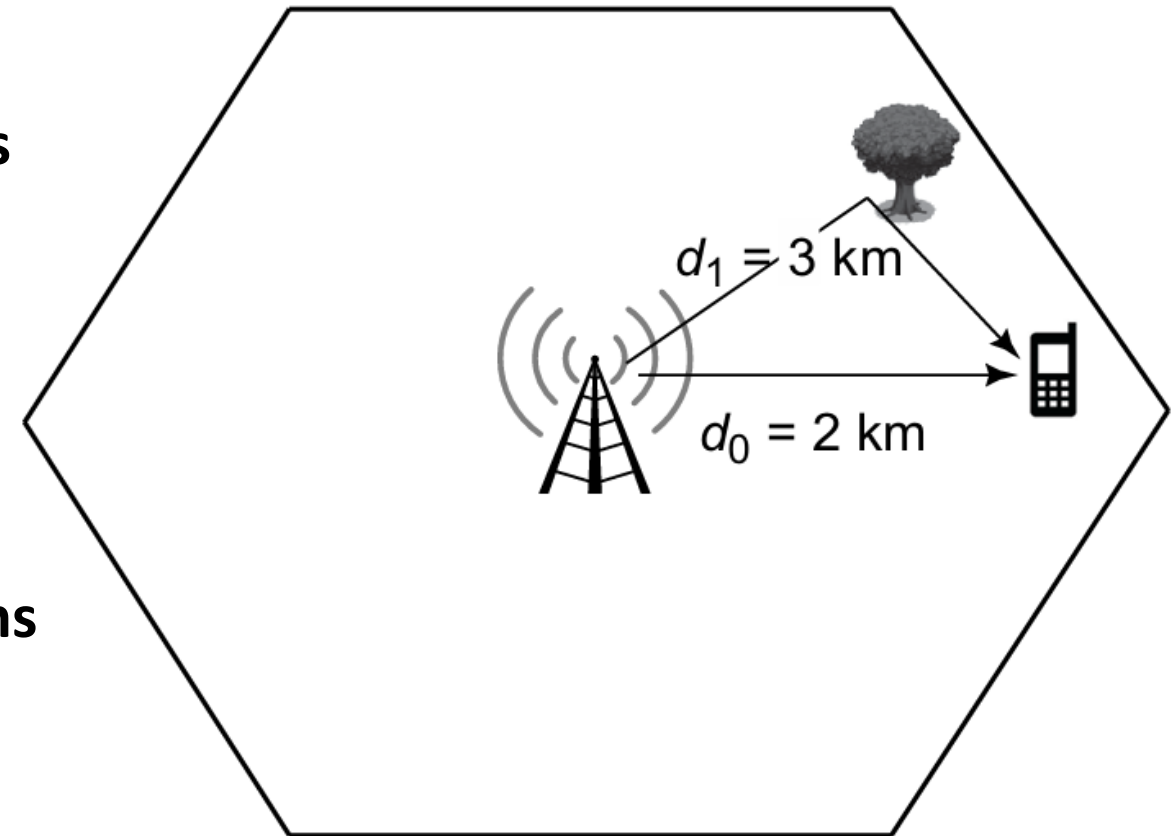
RMS Delay Spread $\sigma_{\tau}^{\text{RMS}}$

τ	dB Gain	g	$a = \sqrt{g}$
$0 \mu s$	−20 dB	0.01	0.1
$1 \mu s$	−10 dB	0.1	0.3162
$3 \mu s$	0 dB	1	1
$5 \mu s$	−10 dB	0.1	0.3162

$$= 0.8573 \mu s$$

2 . Average Delay Spread in Outdoor Cellular Channels

- Typical cell radius: **1 – 5 km**
- Wireless propagation distances: **a few kilometers**
- Signal reaches receiver via **multiple paths**
(direct & scattered)
- Outdoor delay spreads: typically **1 – 3 μ s**
- Indoor distances (~ 10 m): delay spreads **10 – 50 ns**
- Values are **approximate orders**, not exact



3 . Coherence Bandwidth in Wireless Communications

❑ Relation Between Delay Spread and Coherence Bandwidth

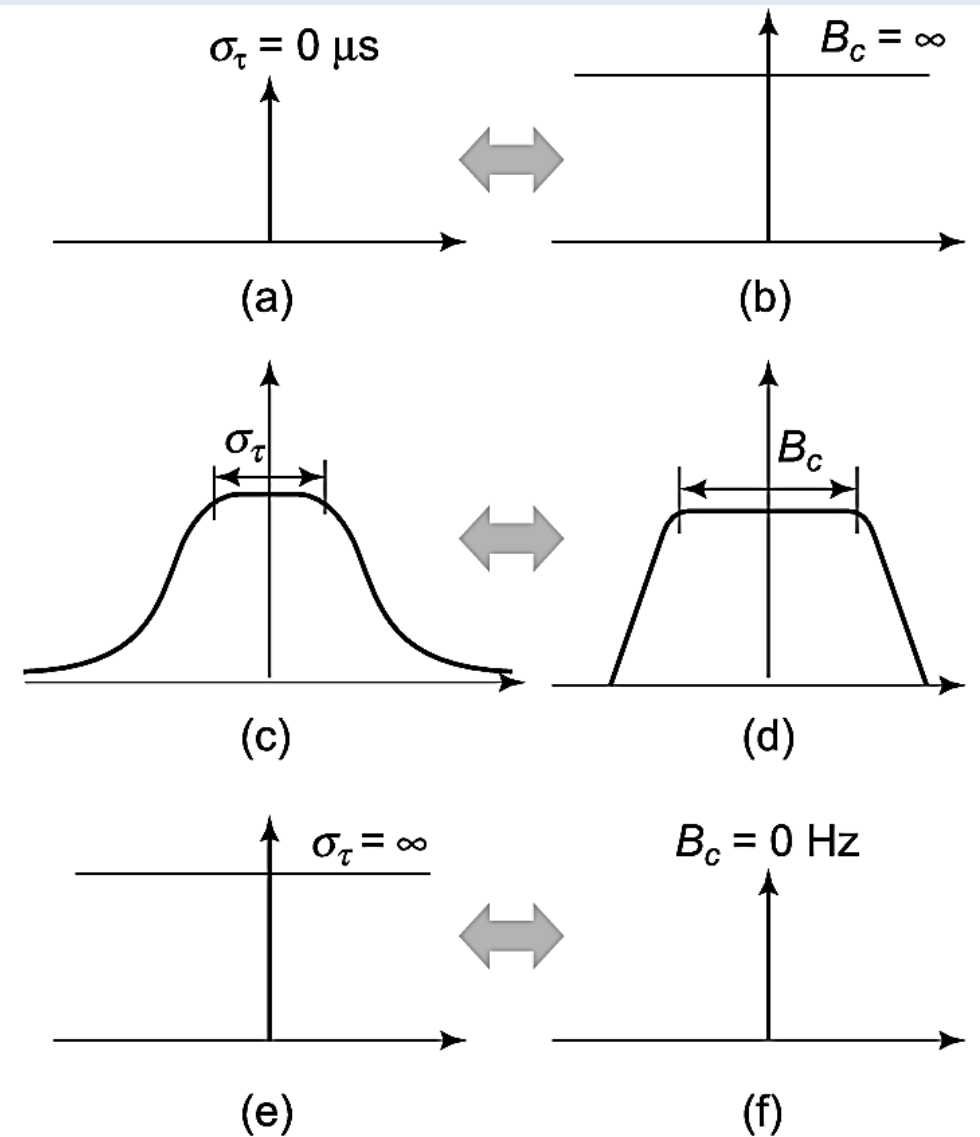
The **delay spread** (σ_τ) and the **coherence bandwidth** (B_c) are inversely related:

$$B_c \approx \frac{1}{\sigma_\tau}$$

- If **delay spread** (σ_τ) is large $\longrightarrow$ **coherence bandwidth** (B_c) is small.
- If **delay spread** (σ_τ) is small $\longrightarrow$ **coherence bandwidth** (B_c) is large.

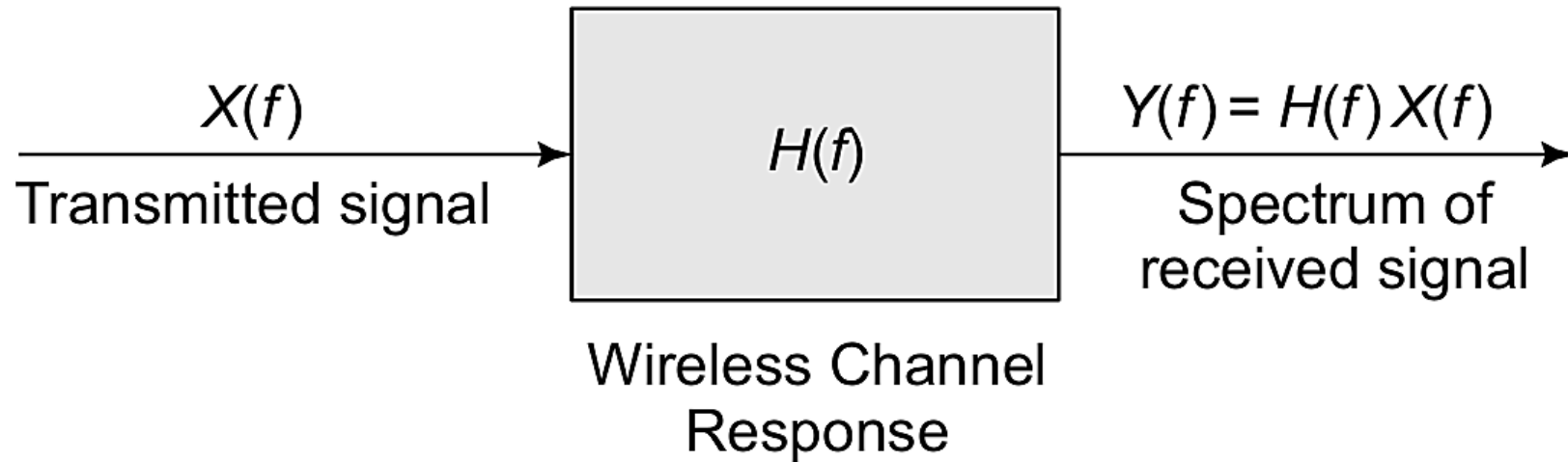
3 . Coherence Bandwidth B_c in Wireless Communications

Coherence bandwidth B_c variation
with delay spread σ_τ

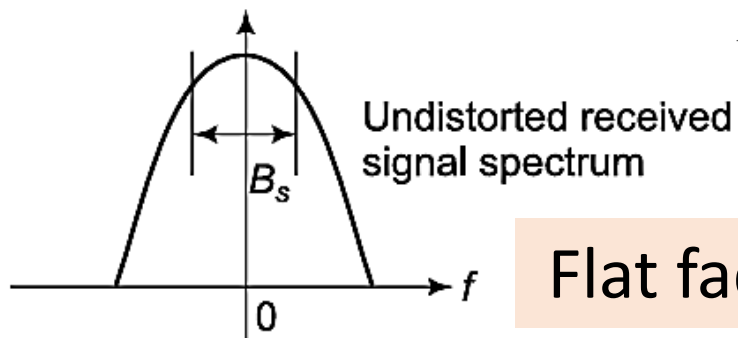
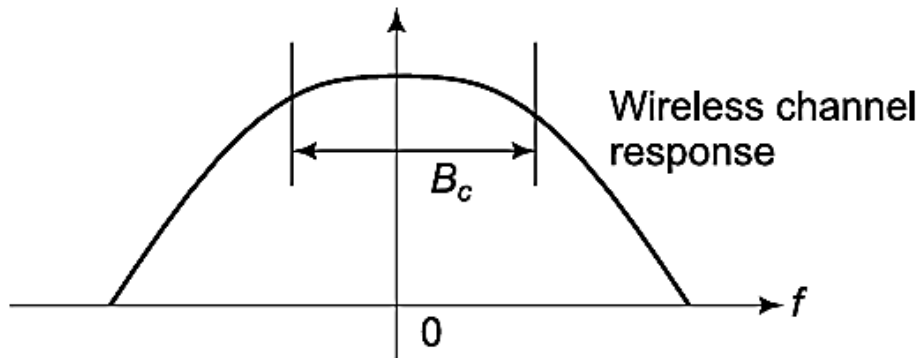
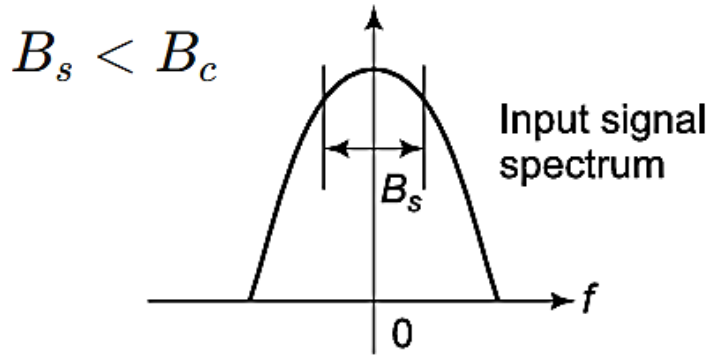


3 . Coherence Bandwidth in Wireless Communications

❑ Linear Input–Output System Model for the Wireless Channel



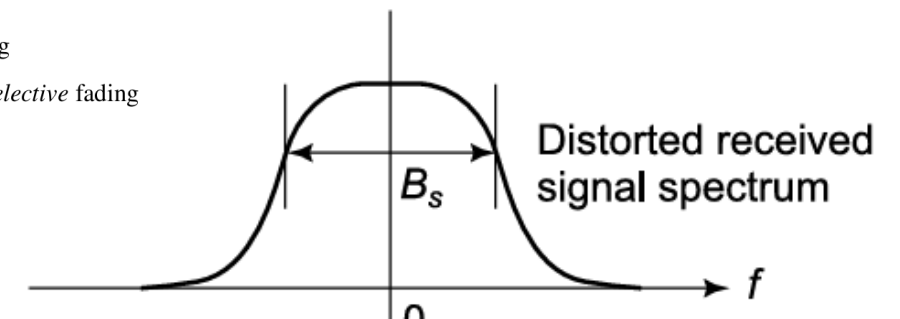
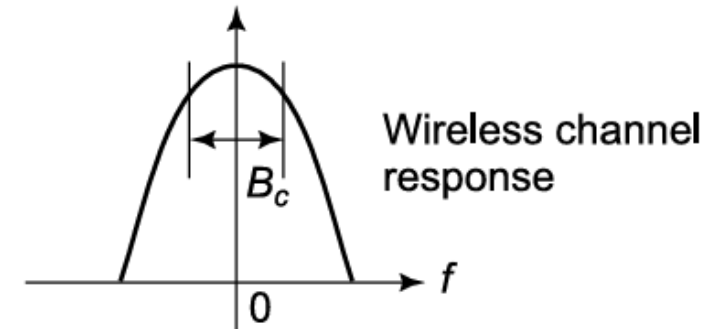
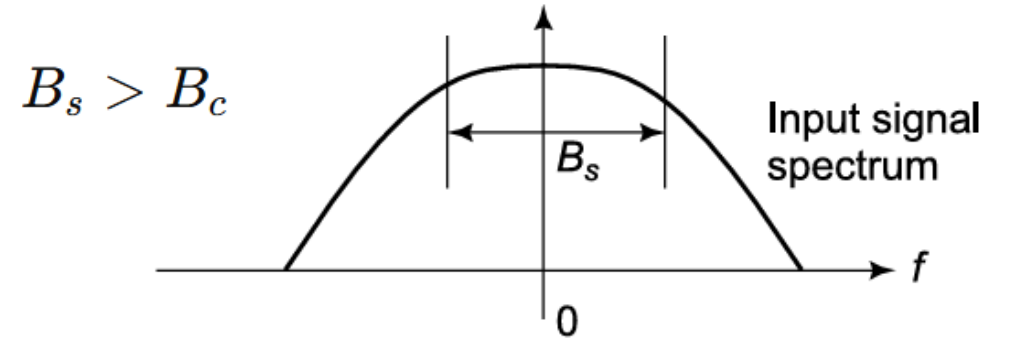
3 . Coherence Bandwidth in Wireless Communications



Flat fading

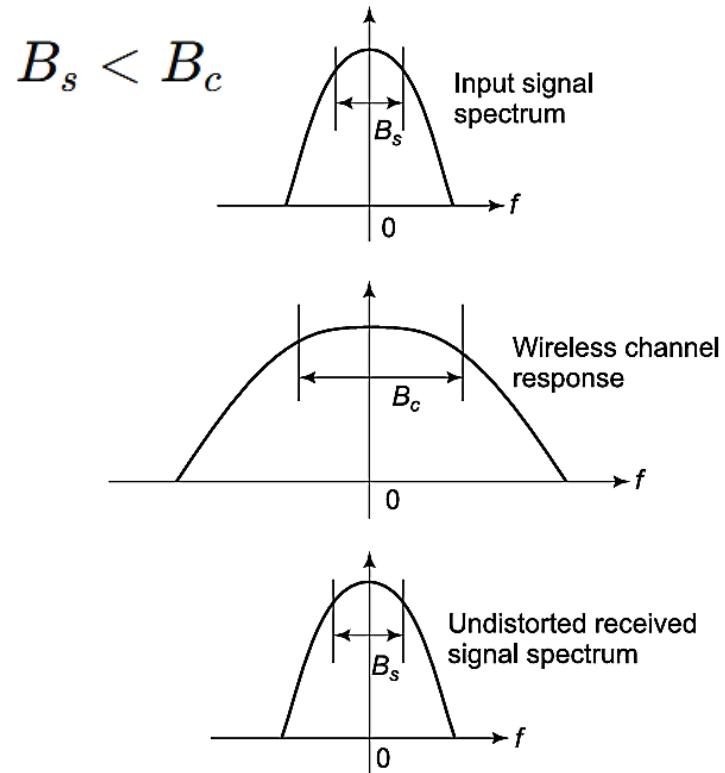
$B_s \leq B_c \Rightarrow$ No distortion in received signal, i.e., flat fading

$B_s \geq B_c \Rightarrow$ Distortion in received signal, i.e., frequency-selective fading

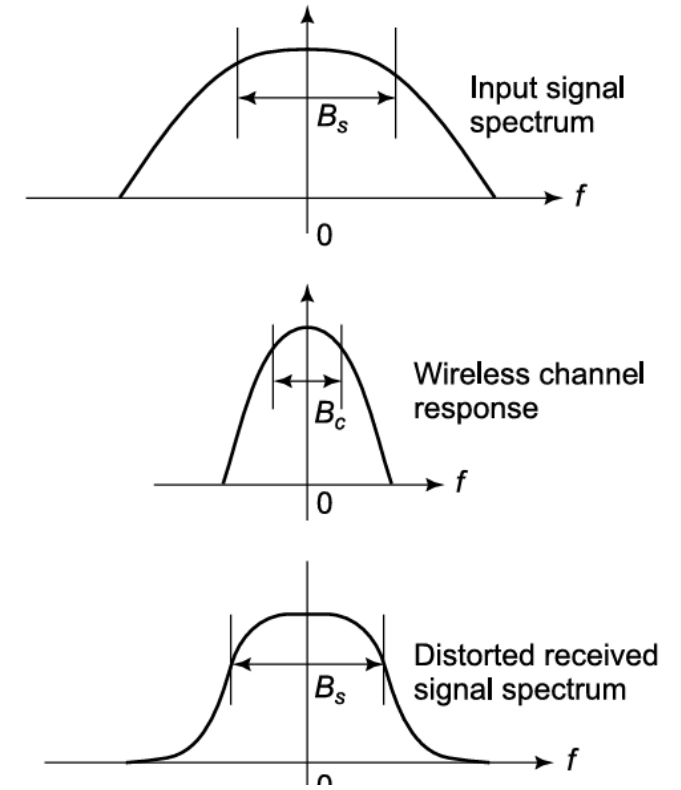


frequency-selective fading

3 . Coherence Bandwidth in Wireless Communications



$$B_s > B_c$$



$B_s \leq B_c \Rightarrow$ No *distortion* in received signal, i.e., *flat fading*

$B_s \geq B_c \Rightarrow$ *Distortion* in received signal, i.e., *frequency-selective fading*

3 . Coherence Bandwidth in Wireless Communications

Tapped-delay line impulse response

$$h(\tau) = \sum_{i=0}^{L-1} a_i \delta(\tau - \tau_i)$$

$$H(f) = \sum_{i=0}^{L-1} a_i \int_0^{\infty} \delta(\tau - \tau_i) e^{-j2\pi f\tau} d\tau.$$

$$H(f) = \int_0^{\infty} h(\tau) e^{-j2\pi f\tau} d\tau$$

$$\int_0^{\infty} \delta(\tau - \tau_i) e^{-j2\pi f\tau} d\tau = e^{-j2\pi f\tau_i}$$

$$H(f) = \int_0^{\infty} \left(\sum_{i=0}^{L-1} a_i \delta(\tau - \tau_i) \right) e^{-j2\pi f\tau} d\tau.$$

$$H(f) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f\tau_i}$$

3 . Coherence Bandwidth in Wireless Communications

Tapped-delay line impulse response

$$H(f) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f \tau_i}$$

1. Value at $f = 0$

$$f = 0 \quad \Rightarrow \quad a_{L-1} e^{-j2\pi \cdot 0 \cdot \tau_{L-1}} = a_{L-1} e^0 = a_{L-1}.$$

So at DC the tap contributes a_{L-1} (no phase shift).

3 . Coherence Bandwidth in Wireless Communications

Tapped-delay line impulse response

$$H(f) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f \tau_i}$$

2. Value at $f = \frac{1}{4\tau_{L-1}}$

Compute the exponent:

$$-j2\pi f \tau_{L-1} = -j2\pi \left(\frac{1}{4\tau_{L-1}} \right) \tau_{L-1} = -j\frac{\pi}{2}.$$

So

$$a_{L-1} e^{-j2\pi f \tau_{L-1}} \Big|_{f=\frac{1}{4\tau_{L-1}}} = a_{L-1} e^{-j\pi/2}.$$

3 . Coherence Bandwidth in Wireless Communications

Tapped-delay line impulse response

$$H(f) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f \tau_i}$$

$$f = 0 \Rightarrow a_{L-1} e^{-j2\pi f \tau_{L-1}} = a_{L-1} e^0 = 1$$

$$f = \frac{1}{4\tau_{L-1}} \Rightarrow a_{L-1} e^{-j2\pi \frac{1}{4\tau_{L-1}} \tau_{L-1}} = a_{L-1} e^{-j\pi \frac{1}{2}} = -ja_{L-1}$$

Interpretation: the magnitude is unchanged but the **phase** has rotated by -90° That is a large phase change

3 . Coherence Bandwidth in Wireless Communications

- The last tap $a_{L-1}e^{-j2\pi f\tau_{L-1}}$ has a phase shift of **90°** when $f = \frac{1}{4\tau_{L-1}}$.
- This is taken as the frequency point where the response begins to change significantly.

So, the **characteristic frequency** is defined as:

$$f_c = \frac{1}{4\tau_{L-1}}.$$

Coherence bandwidth

$$B_c \approx 2f_c = 2 \left(\frac{1}{4\tau_{L-1}} \right) = \frac{1}{2\tau_{L-1}}$$

3 . Coherence Bandwidth in Wireless Communications

Coherence bandwidth $B_c \approx 2f_c = 2 \left(\frac{1}{4\tau_{L-1}} \right) = \frac{1}{2\tau_{L-1}}$

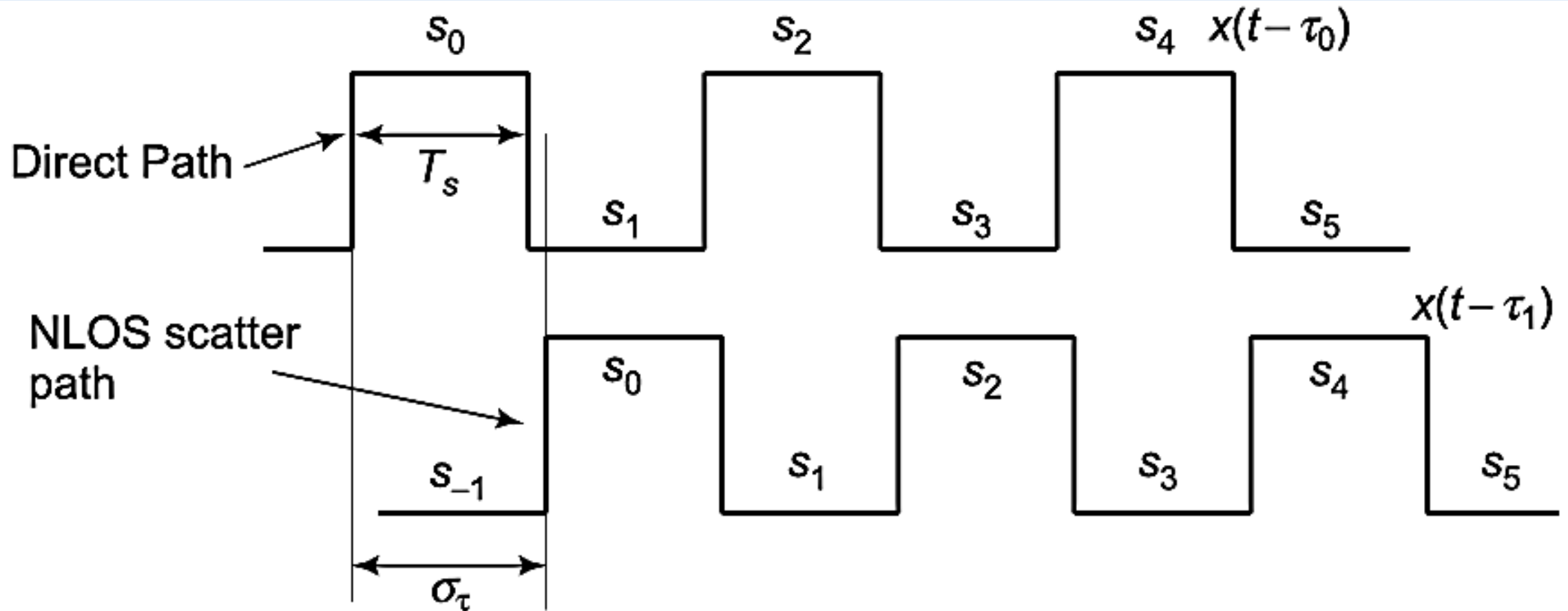
- τ_{L-1} is the **maximum path delay** in the channel.
- This is often called the **maximum delay spread**:

$$\tau_{\max} \approx \tau_{L-1}.$$

So,

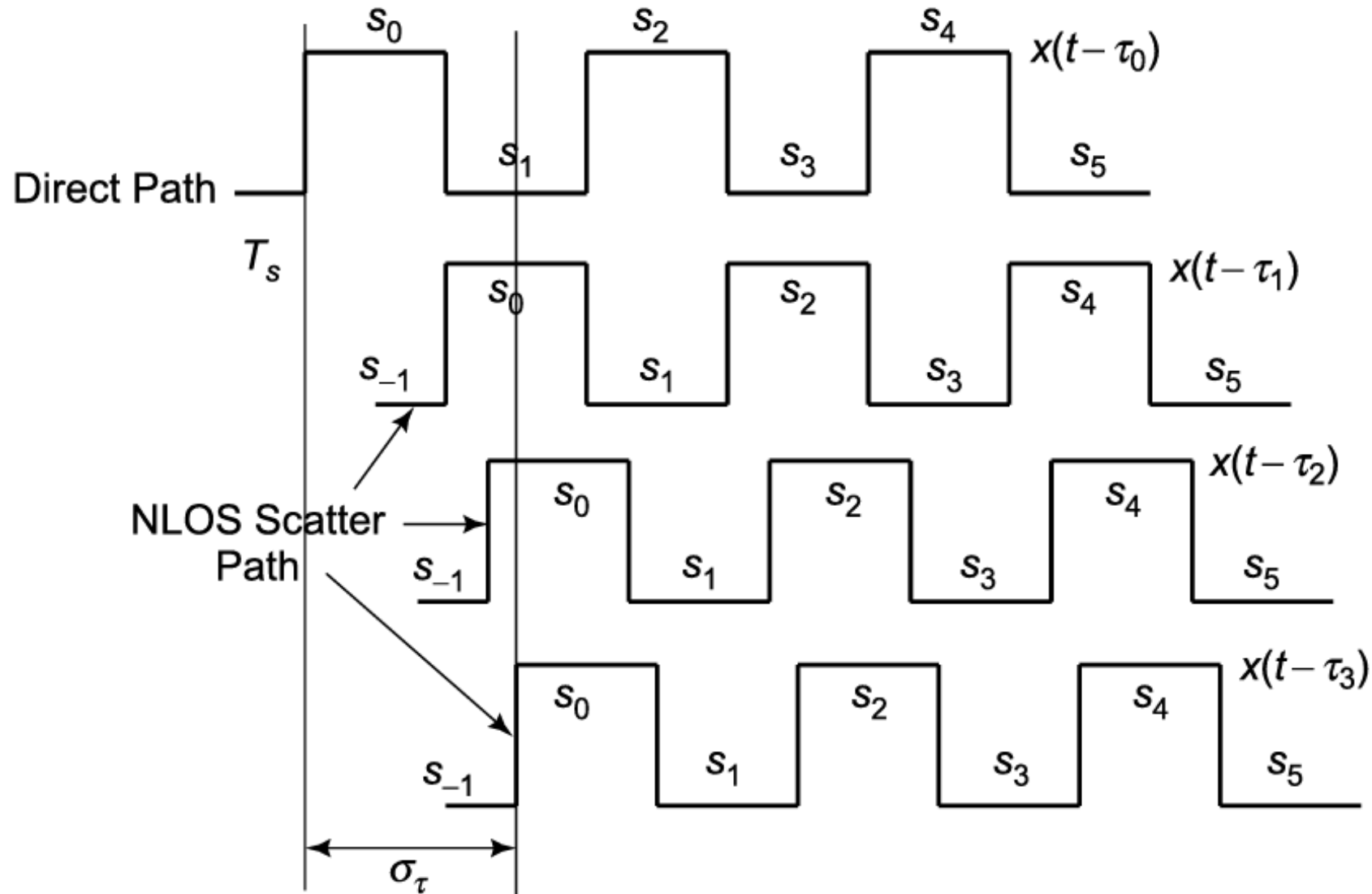
$$B_c \approx \frac{1}{2\tau_{\max}}.$$

4 .Relation Between ISI and Coherence Bandwidth

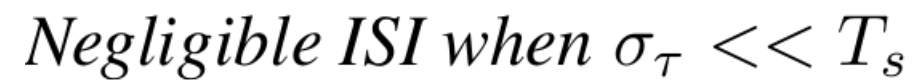


Relation between ISI and delay spread

4 .Relation Between ISI and Coherence Bandwidth



Severe ISI caused by multiple scatter components



4 .Relation Between ISI and Coherence Bandwidth

- **Time-domain view**

- ISI occurs when delay spread $T_d \approx T_s$.
- Condition: $T_d \geq \frac{1}{2}T_s$.
- Symbols overlap $\rightarrow$ distortion.

- **Frequency-domain view**

- Symbol duration: $T_s = \frac{1}{B_s}$.
- Delay spread: $B_c \approx \frac{1}{2T_d}$.
- ISI condition $\rightarrow B_s \geq B_c$.
- Signal bandwidth larger than coherence bandwidth $\rightarrow$ frequency-selective fading.

4 .Relation Between ISI and Coherence Bandwidth

- **Unified insight**
 - ISI (time domain) = Frequency-selective distortion (frequency domain).
 - Both describe the same channel effect.
- **Solution: Equalization**
 - Receiver compensates by applying $1/H(f)$.
 - Restores flat-fading response.
 - Essential for broadband wireless systems (2G, 3G, 4G, 5G).

4 .Relation Between ISI and Coherence Bandwidth

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4 .Relation Between ISI and Coherence Bandwidth

Time Domain (ISI)	Frequency Domain (Coherence Bandwidth)
Delay spread T_d compared to symbol time T_s .	Signal bandwidth B_s compared to coherence bandwidth B_c .
If $T_d \ll T_s \rightarrow$ no ISI (symbols don't overlap).	If $B_s \ll B_c \rightarrow$ flat fading (no distortion).
If $T_d \approx T_s \rightarrow$ ISI occurs (symbols overlap).	If $B_s \geq B_c \rightarrow$ frequency-selective fading (distortion).
ISI condition: $T_d \geq \frac{1}{2}T_s$	Equivalent condition: $B_s \geq B_c$
Several delayed symbols overlap $\rightarrow$ interference grows.	Channel response varies over frequency $\rightarrow$ distortion across band.

4 .Relation Between ISI and Coherence Bandwidth

Consider the 2G Global System for Mobile Communications (GSM) standard with a signal bandwidth of $B_{GSM} = 200$ kHz. Does the GSM signal experience frequency selective or flat fading? Is there inter-symbol interference at the GSM receiver? Answer the same questions as above in the context of the 3G Wideband Code Divison for Multiple Access (WCDMA) standard with a signal bandwidth $B_{WCDMA} = 5$ MHz.

Flat vs. Frequency-Selective Fading in GSM and WCDMA

- Coherence Bandwidth of Cellular Channels:
 $B_c \approx 250$ kHz (outdoor scenario)

WCDMA (3G):

- Signal bandwidth: $B_{WCDMA} = 5$ MHz
- $B_{WCDMA} > B_c \rightarrow$ Frequency-selective fading
- ISI occurs, but handled effectively using RAKE receiver

GSM (2G):

- Signal bandwidth: $B_{GSM} = 200$ kHz
- $B_{GSM} < B_c \rightarrow$ Flat fading
- No Inter-Symbol Interference (ISI) at receiver

5 .Doppler Fading in Wireless Systems

❑ Doppler Shift Principle

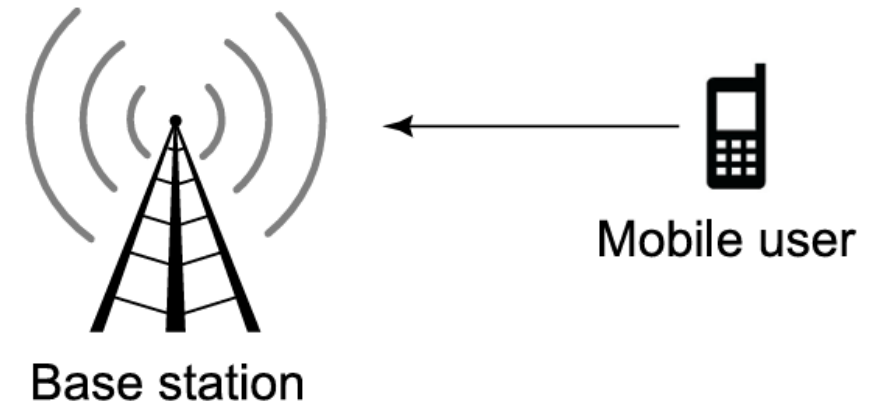
- Change in perceived frequency due to **relative motion** between transmitter & receiver
- Higher frequency → when moving **towards** each other
- Lower frequency → when moving **away**

❑ Cause in Wireless Systems

- Mobility of transceivers (untethered nature) → leads to relative motion
- Unlike **wired systems** → no Doppler effect (fixed medium, no mobility)

❑ Impact on Wireless Communication

- Creates **Doppler fading** (time-varying channel effects)
- Affects signal quality, synchronization, and error rates

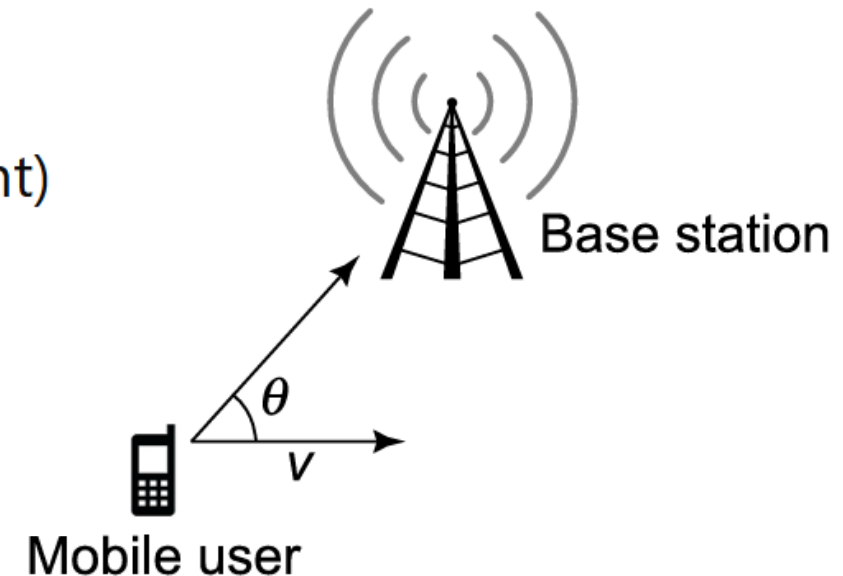


5 .Doppler Fading in Wireless Systems

Doppler Shift in Wireless Systems

$$f_d = \frac{v}{c} \cos \theta \cdot f_c$$

- v = velocity of mobile station
- $c = 3 \times 10^8 \text{ m/s}$ (speed of light)
- f_c = carrier frequency
- θ = angle between direction of motion & line of sight
- f_d increases with velocity v
- Depends on motion angle θ :
 - Max Doppler shift: $\theta = 0, \pi$ (motion along line of sight)
 - Zero Doppler shift: $\theta = \pi/2$ (motion perpendicular)
- Positive shift ($f_{\text{perceived}} > f_c$): when $0 \leq \theta \leq \pi/2$
- Negative shift ($f_{\text{perceived}} < f_c$): when $\pi/2 \leq \theta \leq \pi$



5 .Doppler Fading in Wireless Systems

Consider a vehicle moving at 60 miles per hour at an angle of $\theta = 30^\circ$ with the line joining the base station. Compute the Doppler shift of the received signal at a carrier frequency of $f_c = 1850$ MHz.

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Given:

- Vehicle speed = 60 mph
- Angle = $\theta = 30^\circ$
- Carrier frequency = $f_c = 1850$ MHz
- Speed of light = $c = 3 \times 10^8$ m/s

Convert Speed to m/s

$$\begin{aligned} 60 \text{ mph} &= 60 \times 1.61 \text{ km/h} = 96.6 \text{ km/h} \\ &= 96.6 \times \frac{1000}{3600} = 26.8 \text{ m/s} \end{aligned}$$

Apply Doppler Shift Formula

$$f_d = \frac{v}{c} \cdot f_c \cdot \cos \theta$$

$$f_d = \frac{26.8}{3 \times 10^8} \cdot (1850 \times 10^6) \cdot \cos(30^\circ)$$

$$f_d \approx 143 \text{ Hz}$$

Effective Received Frequency

Since the vehicle is moving towards the BS:

$$f_r = f_c + f_d = 1850 \text{ MHz} + 143 \text{ Hz}$$

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- Carrier frequency = $f_c = 1850$ MHz
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Effective Received Frequency

Since the vehicle is moving towards the BS:

$$f_r = f_c + f_d = 1850 \text{ MHz} + 143 \text{ Hz}$$

6 . Doppler Impact on a Wireless Channel

Impact of Doppler Fading on Multipath Channels

- Multipath channel impulse response (for i-th path):

$$a_i \delta(t - \tau_i)$$

- Vehicle moving with velocity $\mathbf{v}$ at angle θ relative to BS.
- Distance changes continuously $\rightarrow$ delay of each path also changes.
- Initial delay:

$$\tau_i = \frac{d_i}{c}$$

- After time t :

$$\tau_i(t) = \tau_i - \frac{vt}{c} \cos \theta$$

6 . Doppler Impact on a Wireless Channel

Flat-Fading Channel Coefficient

- Flat fading coefficient:

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

- With motion (time-varying delays):

$$h(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_d t}$$

- Where:

$$f_d = \frac{f_c v \cos \theta}{c}$$

6 . Doppler Impact on a Wireless Channel

Interpretation of Doppler Fading

- The term $e^{j2\pi f_d t}$ = time-varying phase shift.
- **Rate of phase variation** = Doppler frequency f_d .
- Doppler → **time-varying channel coefficient**.
- This phenomenon is called **Time Selectivity**.
- **Frequency Selectivity** → attenuation differs across frequencies.
- **Time Selectivity** → attenuation differs across time.
- If both exist → channel is **Doubly Selective**.

7 . Coherence Time of the Wireless Channel

- Similar to **Coherence Bandwidth** in frequency-selective channels.

Coherence Time (T_c):

- Time duration over which the channel can be considered approximately constant.
- Measures **time selectivity** of the wireless channel.

7. Coherence Time of the Wireless Channel

- Time-varying i -th multipath component:

$$a_i(t) = a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_d t}$$

- At $t = 0$:

$$a_i(0) = a_i e^{-j2\pi f_c \tau_i}$$

- At $t = \frac{1}{4f_d}$:

$$a_i\left(\frac{1}{4f_d}\right) = ja_i e^{-j2\pi f_c \tau_i}$$

- $\Rightarrow$ Phase changes by $\pi/2$, so channel changes significantly.

7 . Coherence Time of the Wireless Channel

- Conservative estimate (fastest variation, $\theta = 90^\circ$):

$$T_c = \frac{1}{4f_d^{max}}, \quad f_d^{max} = \frac{v}{c} f_c$$

- Alternative expression (in terms of Doppler spread B_d):

$$T_c = \frac{1}{2B_d}, \quad B_d = 2f_d$$

- **$T_c \approx$ duration where channel is nearly constant.**
- For $t < T_c$: channel can be considered stable.
- For $t > T_c$: channel changes significantly (time selectivity dominates).
- Practical relevance:
 - 3G/4G systems $\rightarrow$ **T_c depends on mobility & carrier frequency.**

7. Coherence Time of the Wireless Channel

Consider the scenario described above in Example 4.5, i.e., a mobile user in a vehicle moving at 60 miles per hour. Compute the coherence time T_c at the carrier frequency $f_c = 1.85$ GHz.

Given

- Speed: 60 mph = 26.8 m/s
- Carrier: $f_c = 1.85$ GHz
- Speed of light: $c = 3 \times 10^8$ m/s
- Use max Doppler (worst case, $\theta = 0^\circ$)

Max Doppler shift

$$f_d^{\max} = \frac{v}{c} f_c = \frac{26.8}{3 \times 10^8} \times (1.85 \times 10^9) \approx \mathbf{165 \text{ Hz}}$$

Doppler spread

$$B_d = 2f_d^{\max} \approx \mathbf{330 \text{ Hz}}$$

Coherence time (using $T_c = \frac{1}{2B_d}$)

$$T_c = \frac{1}{2 \times 330} \approx \mathbf{1.5 \text{ ms}}$$