

Module -2

Code Division Multiple Access (CDMA)

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Outline

Introduction to CDMA

Basic CDMA Mechanism

Fundamentals of CDMA Codes

**Spreading Codes based on Pseudo-Noise (PN)
Sequences**

**Correlation Properties of Random CDMA Spreading
Sequences**

5.1 Introduction to CDMA

- Employed during the 2nd Gen – IS 95 Cellular (Named **cdmaOne**)
- Forms the basis for 3G(WCDMA, HSDPA, HSUPA)
- To understand the concepts in CDMA, it is critical to understand the concept of **multiple access**.
- *Conventional wired Com System – Dedicated Channel,*
- *Wireless Comm- **Share the common channel***

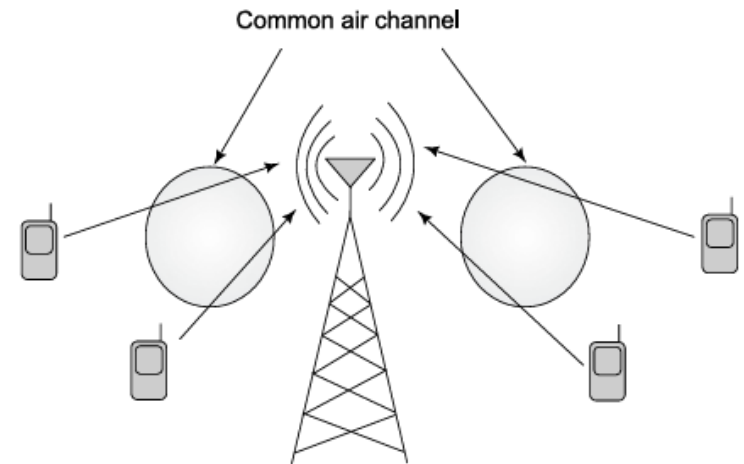


Figure 5.1 Multiple access for wireless cellular networks

Introduction to CDMA

- ***Radio channel - common - wireless frequency bands are limited - device a mechanism for multiple users to access this channel, which is termed as a Multiple Access (MA) technology***
- Multiple users share same frequency band
- Contrast with GSM/FDMA (time/freq slots)
- Distinguished by unique spreading codes
- Enables higher capacity & resistance to interference

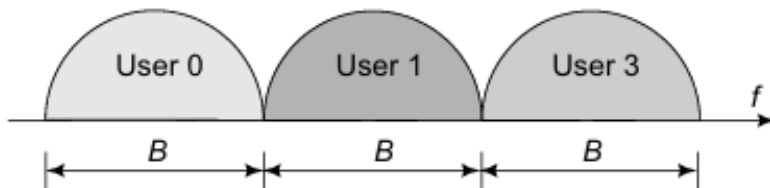


Figure 5.2 Frequency division for multiple access



Figure 5.3 Time division for multiple access

5.2 CDMA Basics

- ***CDMA - is a multiple access technology based on code division. In other words, different users are multiplexed using different codes***
- Spreading codes assigned to each user
- Data bits multiplied with spreading sequence
$$\begin{aligned}x_0 &= a_0 \times [1, 1, 1, 1] & x_1 &= a_1 \times [1, -1, -1, 1] \\ &= [a_0, a_0, a_0, a_0] & &= [a_1, -a_1, -a_1, a_1]\end{aligned}$$
- Thus, the users of the different signals are multiplexed over the common wireless channel employing different codes. Hence, this is termed Code Division for Multiple Access

5.2 CDMA Basics

The **key operations in CDMA can be summarized as follows.**

1. Multiplying or modulation the symbols of the different users with the corresponding assigned unique code
2. Combining or adding the code-modulated signals of all the users to form the composite signal.
3. Finally, correlation of the composite received signal x at each user with the corresponding code of the user to recover the transmitted symbol.

5.2 CDMA Basics

$$\begin{aligned}
 x_0 &= a_0 \times [1, 1, 1, 1] \\
 &= [a_0, a_0, a_0, a_0]
 \end{aligned} \tag{5.1}$$

$$\begin{aligned}
 x_1 &= a_1 \times [1, -1, -1, 1] \\
 &= [a_1, -a_1, -a_1, a_1]
 \end{aligned} \tag{5.2}$$

$$x = x_1 + x_2 = [(a_0 + a_1), (a_0 - a_1), (a_0 - a_1), (a_0 + a_1)] \tag{5.3}$$

$$\begin{array}{rcccc}
 & a_0 + a_1 & a_0 - a_1 & a_0 - a_1 & a_0 + a_1 \\
 \times & 1 & 1 & 1 & 1 \\
 \hline
 & (a_0 + a_1) + (a_0 - a_1) + (a_0 - a_1) + (a_0 + a_1) = 4a_0 & & &
 \end{array} \tag{5.4}$$

$$\begin{array}{rcccc}
 & a_0 + a_1 & a_0 - a_1 & a_0 - a_1 & a_0 + a_1 \\
 \times & 1 & -1 & -1 & 1 \\
 \hline
 & (a_0 + a_1) - (a_0 - a_1) - (a_0 - a_1) + (a_0 + a_1) = 4a_1 & & &
 \end{array} \tag{5.5}$$

5.3 Fundamentals of CDMA Codes

- Orthogonal Property - recover the symbols of the different users from the composite signal.

$$r_{01} = \sum_{k=0}^3 c_0(k) c_1(k)$$

1. For a user with symbol rate $R = 1$ kbps, the symbol time is $T = 1$ ms, and the required transmission bandwidth is $B = 1/T = 1$ kHz.
2. Each symbol is transmitted with its corresponding spreading code (e.g., $[1, 1, 1, 1]$), which requires 4 chips per symbol when $N = 4$.
3. To maintain the symbol rate of 1 kbps, the chip time must be set as $T_c = T/N = 0.25$ ms, leading to a system bandwidth of $B_{CDMA} = 1/T_c = 4$ kHz.
4. Increasing the spreading code length N increases the required bandwidth proportionally, effectively **spreading** the signal and making CDMA a **spread spectrum (or wideband) system**.

5.3 Fundamentals of CDMA Codes

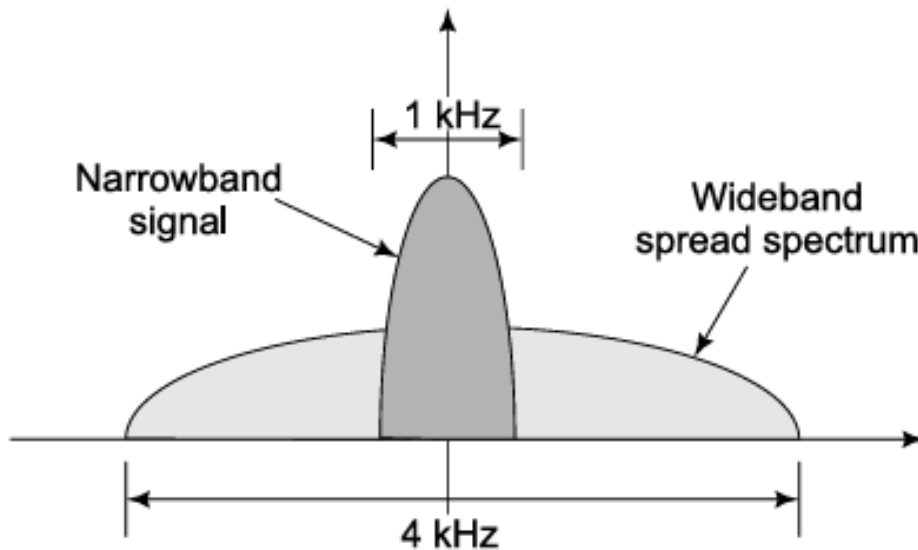


Figure 5.4 *Spread spectrum communication*

Symbol Rate – 1kbps

$T_s = 1\text{ms}$

$B = 1\text{KHz}$

Chip Time $T_c = T/N = 0.25\text{ms}$

$B_{\text{CDMA}} = 1/T_c = 4\text{KHz}$

5.3 Fundamentals of CDMA Codes

- ❑ With a spreading sequence of length N , there are N **orthogonal codes**, allowing N **users to be multiplexed simultaneously**.
- ❑ Although the bandwidth increases by a factor of N , the **parallel transmission of N users compensates for it**, so the **spectral efficiency remains unaffected** (as shown in Fig. 5.5).

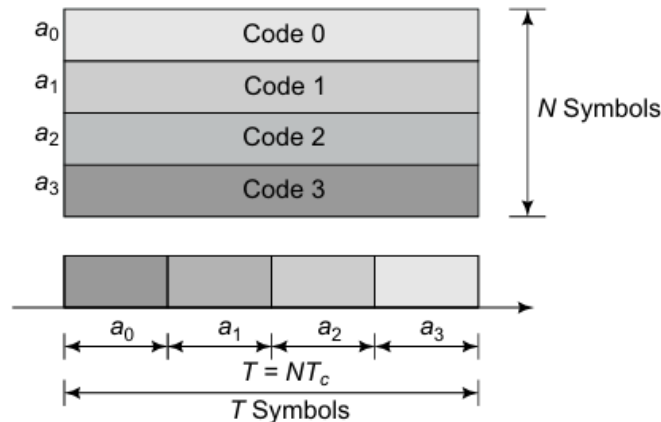


Figure 5.5 Parallel transmission of N symbols over N codes in CDMA over time interval $T = NT_c$ (above) and comparison with transmission of N symbols in time $T = NT_c$ in a conventional single carrier or time division system

5.4 Spreading Codes based on Pseudo-Noise(PN) Sequences

- ❑ Code looks like a random sequence – Not actually
- ❑ A pseudo-noise (PN) sequence – only resembles a noise sequence, but is not actually a noise sequence
- ❑ One method – Linear Feedback Shift Register (LFSR)

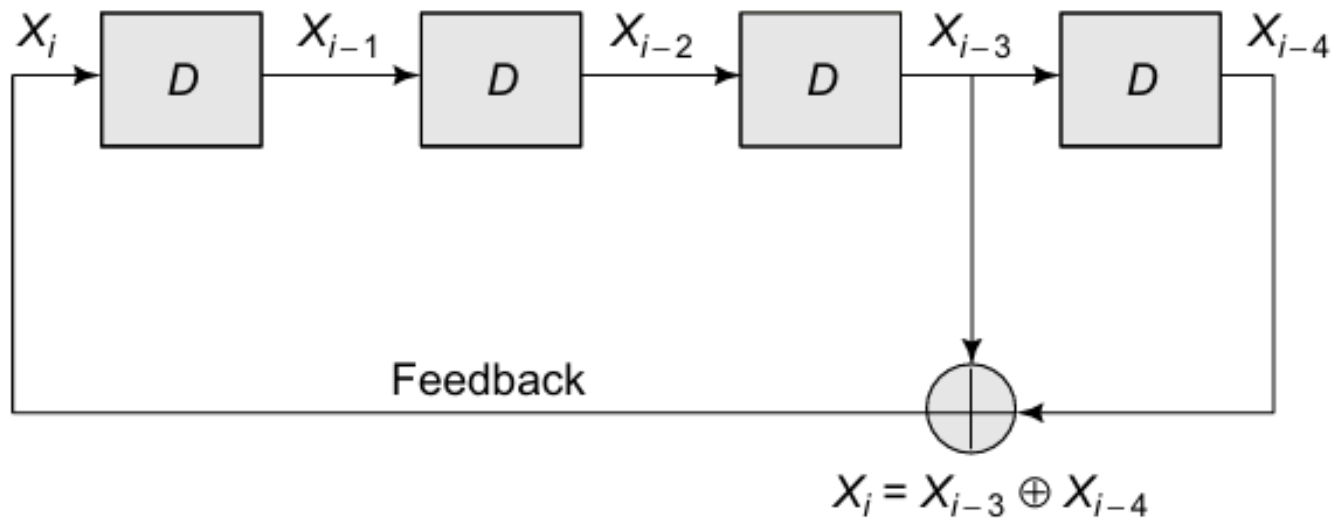


Figure 5.6 *Linear feedback shift register*

5.4 Spreading Codes based on Pseudo-Noise(PN) Sequences

1. The system uses a **shift register** with **4 delay elements** ($D = 4$), where the feedback is formed by XORing the outputs of the 3rd and 4th delays ($X_{i-3} \oplus X_{i-4}$) and feeding it back as the new input X_i .
2. The governing equation is:

$$X_i = X_{i-3} \oplus X_{i-4}$$

Since this is a **linear relation with feedback**, the system is called a **Linear Feedback Shift Register (LFSR)**.

3. The register's next state depends on its previous values ($X_{i-1}, X_{i-2}, X_{i-3}, X_{i-4}$), making it a **state-based system**. For example, with initialization $X_{i-1} = X_{i-2} = X_{i-3} = X_{i-4} = 1$, the sequence of X_i values can be generated accordingly.

1111, 0111, 0011, 0001, 1000, 0100, 0010, 1001, 1100, 0110, 1011, 0101, 1010, 1101, 1110,

Does not goes to All-Zero State “0000”



5.4 Spreading Codes based on Pseudo- Noise(PN) Sequences

X_{i-1}	X_{i-2}	X_{i-3}	X_{i-4}	X_i
1	1	1	1	0
0	1	1	1	0
0	0	1	1	0
0	0	0	1	1
1	0	0	0	0
0	1	0	0	0
0	0	1	0	1
1	0	0	1	1
1	1	0	0	0
0	1	1	0	1
1	0	1	1	0
0	1	0	1	1
1	0	1	0	1
1	1	0	1	1
1	1	1	0	1

5.4 Spreading Codes based on Pseudo-Noise(PN) Sequences

- ❖ For instance, for the above LFSR, the maximum-length PN sequence is the sequence of outputs X_{i-4} given as

PN Sequence = 1 1 1 1 0 0 0 1 0 0 1 1 0 1 0

- ❖ We use standard BPSK Modulation Map ($1 \rightarrow -1$ & $0 \rightarrow +1$)

PN sequence = -1 -1 -1 -1 +1 +1 +1 -1 +1 +1 -1 -1 +1 -1 +1

5.4 Spreading Codes based on Pseudo-Noise(PN) Sequences – **Properties**

Property 1- Balance Property : Counting the number of -1 and +1 chips in the sequence, (-1 will be more than +1)

Property 2 - Run-Length Property: String of continuous values, 1/8 – Length 3, 1/4 - Length 2, 1/2 - Length 1

Property 3 – Correlation Property:

$$r_{00}(n_o) = \frac{1}{N} \sum_{i=0}^{N-1} = \begin{cases} 1 & \text{if } n_o = 0 \\ -\frac{1}{N} & \text{otherwise} \end{cases}$$



5.4 Spreading Codes based on Pseudo-Noise(PN) Sequences – **Properties**

Property 1- Balance Property :

PN sequence = $-1 -1 -1 -1 +1 +1 +1 -1 +1 +1 -1 -1 +1 -1 +1$

1. A PN sequence of length $2^D - 1$ has almost equal number of +1's and -1's.
2. The difference between the number of +1's and -1's is at most one.
3. Example: In the above sequence of length 15, the number of -1's = 8 and the number of +1's = 7.



5.4 Spreading Codes based on Pseudo-Noise(PN) Sequences – **Properties**

Property2 - Run-Length Property :

- ❖ A run is a string of continuous values. In the PN sequence, there are a total of 8 runs like: $-1, -1, -1, +1$.

Among the 8 runs, it is observed that $\frac{1}{8}$ of the runs are of length 3, $\frac{1}{4}$ of the runs are of length 2, and $\frac{1}{2}$ of the runs are of length 1. This characteristic is known as the **run-length property** of PN sequences.

5.4 Spreading Codes based on Pseudo-Noise(PN) Sequences – **Properties**

Property 3 – Correlation Property :

❖ The correlation property is one of the most important properties of PN sequences

PN sequence = -1 -1 -1 -1 +1 +1 +1 -1 +1 +1 -1 -1 +1 -1 +1

Self -Correlation

$$\begin{aligned} r_{00}(0) &= \frac{1}{N} \sum_{i=0}^{N-1} c_0(n) c_0(n) \\ &= \frac{1}{N} \sum_{i=0}^{N-1} 1 \\ &= \frac{1}{N} \times N = 1 \end{aligned}$$

Delayed Sequence - Correlation

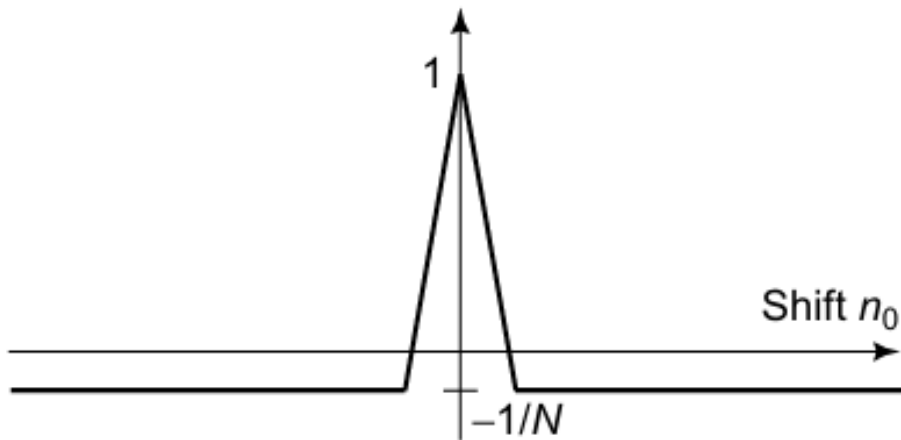
$$\begin{aligned} \text{PN Sequence} &= -1 + 1, -1 -1 -1 -1 +1 +1 \\ &\quad +1 -1 +1 +1 -1 -1 +1 \end{aligned} \tag{5.7}$$

$$\begin{aligned} r_{00}(2) &= \frac{1}{N} \sum_{i=0}^{N-1} c_0(n) c_0(n-2) \\ &= \frac{1}{15} (1 -1 +1 +1 -1 -1 +1 -1 +1 -1 -1 -1 -1 +1 +1) \\ &= \frac{1}{15} \times (-1) = -\frac{1}{15} \\ &= -\frac{1}{N} \end{aligned}$$

5.4 Spreading Codes based on Pseudo-Noise(PN) Sequences – Properties

Property 3 – Correlation Property :

❖ This auto correlation property of the PN sequence, i.e., of the sequence with a delayed version of itself, is shown pictorially



$$r_{00}(n_o) = \frac{1}{N} \sum_{i=0}^{N-1} = \begin{cases} 1 & \text{if } n_o = 0 \\ -\frac{1}{N} & \text{otherwise} \end{cases}$$

Figure 5.7 Autocorrelation of PN sequence

5.7 CDMA- Advantages

- ❖ **Jammer Margin**
- ❖ **Graceful Degradation**
- ❖ **Universal Frequency Reuse**
- ❖ **Multipath Diversity and Rake Receiver**

5.7 CDMA- Advantages

❖ Advantage 1:

Jammer Margin

- ❖ An advantage over conventional cellular system is Jammer suppression
- ❖ Malicious user - very high power –disruption of communication links

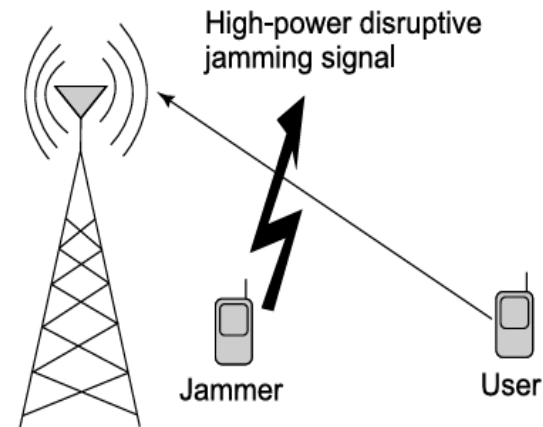


Figure 5.8 Disruption by jammer in wireless communication

5.7 CDMA- Advantages

Consider a communication system in which the signal $x(n)$ of the power P is received in the presence of additive white Gaussian noise $w(n)$ of power σ_w^2 . The baseband system model for this communication system can be expressed as

$$y(n) = x(n) + w(n)$$

Hence, the SNR at the receiver is $\text{SNR} = \frac{P}{\sigma_w^2}$. However, in the presence of a jamming signal $x_j(n)$ of power P_j , the received signal $y(n)$ is

$$y(n) = x(n) + x_j(n) + w(n)$$

Signal-to-interference-noise power ratio (SINR) can be calculated

$$\text{SINR} = \frac{P}{P_j + \sigma_w^2}$$

5.7 CDMA- Advantages

- Consider now a CDMA system in which the transmitted signal $x(n)$ is a spread-spectrum signal.

$$\text{SINR} = \frac{P}{\frac{P_j}{N} + \frac{\sigma_w^2}{N}}$$

- Thus, it can be seen that the jamming power P_j is suppressed by a factor of N .
- Moreover, as the spreading factor N increases, the jammer suppression increases, minimizing the impact of the jammer on the communication system. This is termed **jammer suppression in CDMA systems (jammer margin)**

5.7 CDMA- Advantages

❖ Advantage 2: Graceful Degradation

- ❖ It allows for much more efficient interference management which ultimately leads to universal frequency reuse and higher spectral efficiency.
- ❖ Adding one more user with index K+1

The SINR of the user 0 now changes to

$$\begin{aligned}\text{SINR} &= \frac{P_0}{\frac{P_1}{N} + \frac{P_2}{N} + \dots + \frac{P_K}{N} + \frac{P_{K+1}}{N} + \frac{\sigma_n^2}{N}} \\ &= N \times \frac{P_0}{\sum_{k=0}^{K+1} P_k + \sigma_n^2}\end{aligned}$$

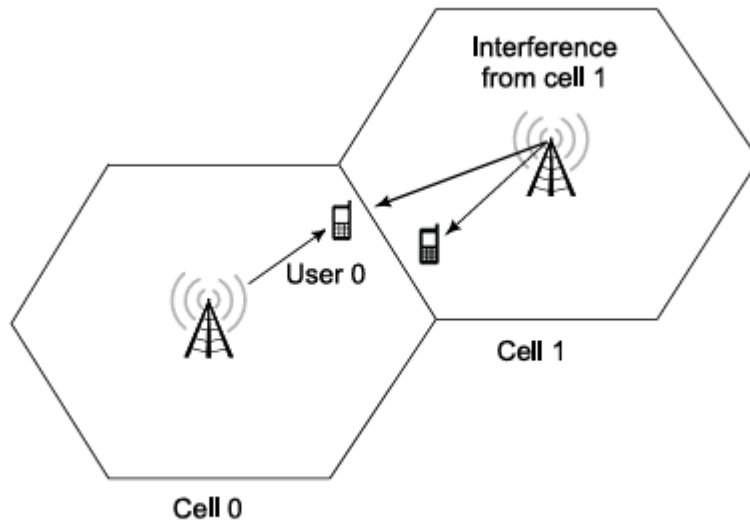
5.7 CDMA- Advantages

❖ Graceful Degradation

- ❖ Additional interference caused by this new user is shared amongst all the existing users in the system leading to interference distribution - leads to a graceful degradation of the SINR at each user.
- ❖ ***This is termed the graceful degradation property of CDMA systems***

5.7 CDMA- Advantages

❖ Advantage3: Universal Frequency Reuse



$$y_0(n) = \underbrace{x_0(n)}_{\text{Signal}} + \underbrace{x_1(n)}_{\text{Interferer from } C_1} + \underbrace{w(n)}_{\text{Noise}}$$

Figure 5.9 Intercell interference for the user 0 on the cell edge

5.7 CDMA- Advantages

❖ Advantage 3: Universal Frequency Reuse

- **Conventional Cellular (GSM) Limitation**
- In a typical 1G/2G system such as GSM, only a fraction of the available frequency bands can be allocated to each cell (e.g., reuse factor 7).
- This reduces capacity since each cell uses only part of the spectrum
- **Frequency Reuse Factor in GSM**
- GSM uses a frequency reuse factor of **1/7**, meaning only one-seventh of the total available frequency bands are assigned to a given cell

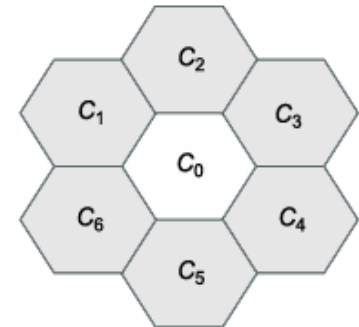


Figure 5.10 Grid or lattice of hexagonal cells

5.7 CDMA- Advantages

❖ Advantage 3: Universal Frequency Reuse

- **Universal Frequency Reuse in CDMA**
- CDMA allows **all frequency bands to be used in every cell**, also called **frequency reuse factor 1** or **universal frequency reuse Capacity Gain**
- Compared to GSM, which uses only 1/7 frequency in each cell, CDMA uses the **entire frequency band** in every cell.
- This increases **spectral efficiency and capacity by a factor of 7**, giving CDMA a much higher capacity than conventional cellular standards

$$\text{SINR} = \frac{P_0}{\frac{P_1}{N} + \frac{\sigma_w^2}{N}}$$

5.7 CDMA- Advantages

❖ Advantage 4: Multipath Diversity and Rake Receiver

- ❖ Another important advantage of CDMA is its ability to achieve diversity gain via multipath scatter components. This is termed **multipath diversity** and is achieved through coherent combining of the multipath-signal components employing a rake receiver.

$$y(n) = h(0)x(n) + h(1)x(n-1) + \dots + h(L-1)x(n-L+1) + w(n)$$

$$= \sum_{l=0}^{L-1} h(l)x(n-l) + w(n)$$

Consider now a CDMA signal in which $x(n) = a_0 c_0(n)$.

$$y(n) = \sum_{l=0}^{L-1} h(l)a_0 c_0(n-l) + w(n)$$

5.7 CDMA- Advantages

❖ Advantage 4: Multipath Diversity and Rake Receiver

- **Correlation with Spreading Codes**
- The RAKE receiver correlates the received signal with the spreading sequence at different delays.
- This allows separation and extraction of individual multipath components
- By correlating with delayed spreading codes, distinct multipath components $d(0), d(1), \dots, d(L-1)$ can be obtained.
- Each component corresponds to a specific delayed path, which can be independently processed -

Orthogonal Frequency Division Multiplexing(OFDM)



7.1 Introduction

- ❖ Orthogonal Frequency Division Multiplexing (OFDM) is the core technology behind **4G wireless systems** like LTE and WiMAX.
- ❖ OFDM supports **very high data rates (100+ Mbps)**, making it suitable for broadband wireless communication.
- ❖ Apart from cellular (4G), OFDM is also used in **wireless LAN standards** such as IEEE 802.11a/g/n (Wi-Fi)
- ❖ The central idea of OFDM is **multicarrier transmission**, where available bandwidth is divided into multiple orthogonal subcarriers to increase efficiency and reduce interference

7.2 Motivation & Multicarrier Concept

1. Single-carrier system limitation

- For a single-carrier system of bandwidth B , symbol time is $T = 1/B$.
- Symbol rate = B .
- Symbols are transmitted sequentially, one every T seconds .

2. Multicarrier division

- The total bandwidth B is divided into N sub-bands of bandwidth B/N .
- Each sub-band corresponds to one subcarrier with center frequency $f_i = iB/N$

3. Parallel data streams

- Instead of one stream at rate B , N parallel streams are transmitted at lower rate B/N each.
- This reduces the symbol rate per subcarrier, increasing robustness to ISI (Inter-Symbol Interference).

4. Orthogonality of subcarriers

- Subcarriers are spaced exactly B/N apart, ensuring orthogonality.
- This prevents interference among them, even though they overlap in frequency.

7.2 Motivation & Multicarrier Concept

the signal $s_i(t)$ corresponding to the i^{th} subcarrier is given as

$$s_i(t) = X_i e^{j2\pi f_i t} = X_i e^{j2\pi i \frac{B}{N} t}$$

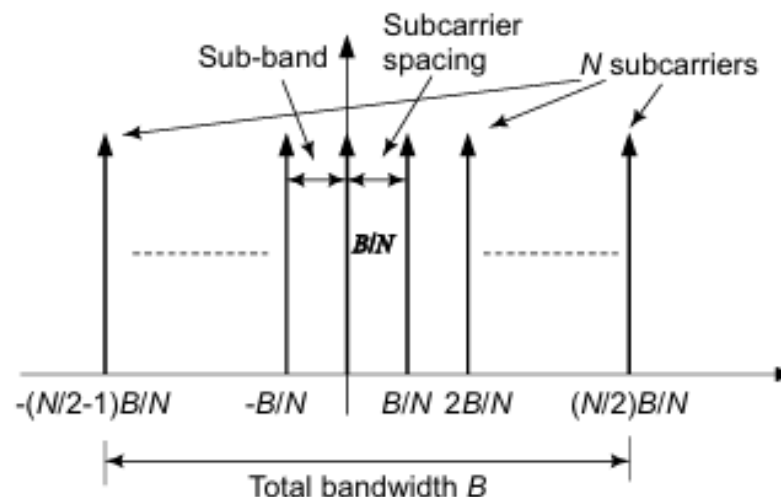


Figure 7.1 Multi-carrier concept

Example case

- For $B = 256$ kHz with $N = 64$, sub-band width = 4 kHz.
- Each subcarrier transmits its own modulated symbol, forming a multi-carrier system .

7.2 Motivation & Multicarrier Concept

- ❖ These signals are then superposed at the transmitter to form the composite signal $s(t)$ given as

$$s(t) = \sum_i s_i(t) = \sum_i X_i e^{j2\pi \frac{iB}{N} t}. \quad (7.2)$$

- ❖ At the receiver, the individual data streams have then to be isolated from the composite signal $s(t)$ above, by Fourier Series

$$\begin{aligned} f_0 \int_0^{T_0} y(t) \left(e^{j2\pi f_l t} \right)^* dt &= \frac{B}{N} \int_0^{\frac{N}{B}} \left(\sum_i X_i e^{j2\pi i \frac{B}{N} t} \right) e^{-j2\pi l \frac{B}{N} t} dt \\ &= \frac{B}{N} \sum_i \int_0^{\frac{N}{B}} X_i e^{j2\pi(i-l) f_0 t} dt \end{aligned}$$

7.2 Motivation & Multicarrier Concept

$$\begin{aligned}
 &= \underbrace{\frac{B}{N} \int_0^{\frac{N}{B}} X_l dt}_{i=l} + \frac{B}{N} \sum_{i \neq l} \int_0^{\frac{N}{B}} X_i e^{j2\pi(i-l)f_0 t} dt \\
 &= X_l + \frac{B}{N} \sum_{i \neq l} X_i \underbrace{\int_0^{\frac{N}{B}} e^{j2\pi(i-l)f_0 t} dt}_{=0} \\
 &= X_l
 \end{aligned}$$

where we have used the fact that $\int_0^{T_0} e^{j2\pi(i-l)f_0 t} dt = 0$ for $i \neq l$, since this is basically integrating a sinusoid of frequency $(i-l)f_0$, which is a multiple of the fundamental frequency f_0 over the period T_0 . Therefore, since there are an integer number of cycles of the sinusoid of frequency $(i-l)f_0$, this integral is 0. In fact, this basically implies that the different sinusoids $e^{j2\pi i f_0 t}$ and $e^{j2\pi l f_0 t}$ are *orthogonal*. It is this key property of *orthogonality* which helps extract the different streams X_i modulated over the different subcarriers. This property of orthogonality can be summarized as

$$\int_0^{N/B} e^{j2\pi(i-l)\frac{B}{N}t} dt = \begin{cases} 0 & i \neq l \\ \frac{N}{B} & i = l \end{cases}$$

7.2 Multicarrier Transmission

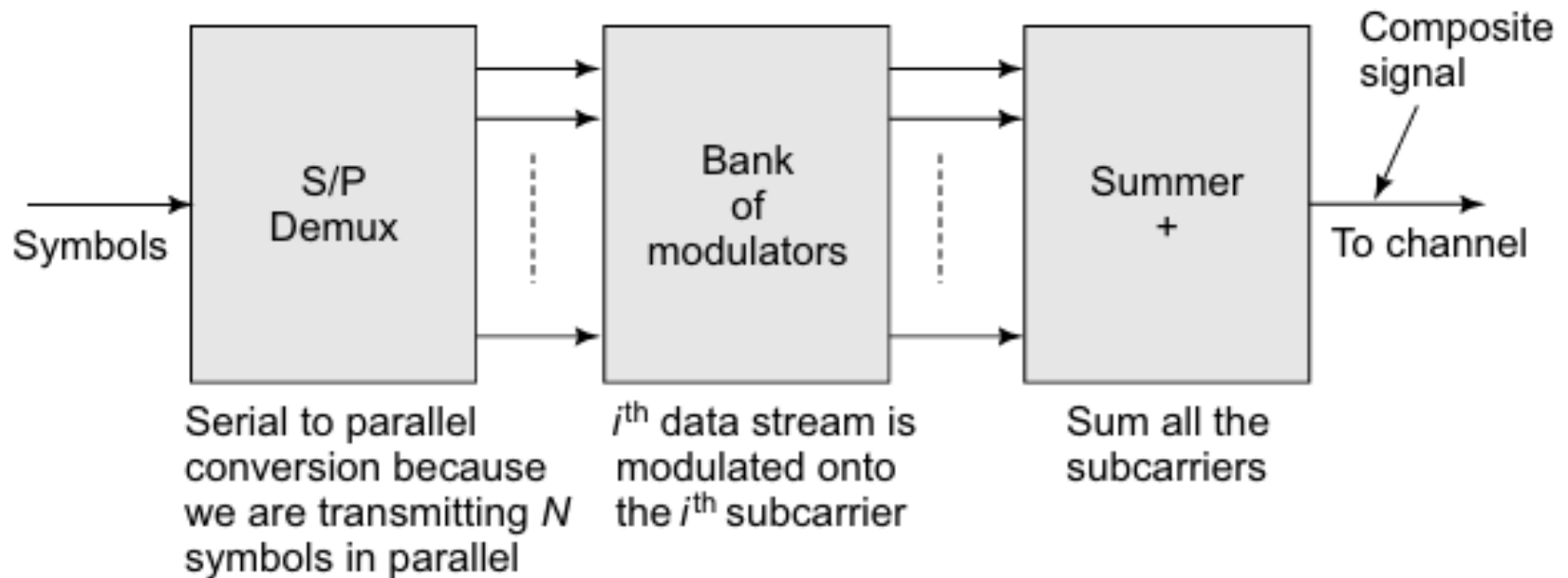


Figure 7.2 *Multicarrier modulation transmitter*

7.2 Multicarrier Transmission

- **Serial-to-Parallel Conversion**

- The incoming serial bit stream is split into N parallel data streams using a **Serial-to-Parallel (S/P) Demultiplexer**.
- This allows multiple symbols to be transmitted simultaneously.

- **Assignment to Subcarriers**

- Each parallel data stream is assigned to a distinct **subcarrier**.
- The i th data stream is mapped to the i th subcarrier frequency.

- **Bank of Modulators**

- Each subcarrier is individually modulated (e.g., QPSK, QAM) with its corresponding data stream.
- This produces modulated signals at different subcarrier frequencies.

7.2 Multicarrier Transmission

- **Summation of Subcarriers**
 - All the modulated subcarrier signals are summed together in the **Summer** block.
 - The result is a composite multicarrier signal containing all subcarriers.
- **Transmission to Channel**
 - The composite signal, which now carries all parallel streams simultaneously, is transmitted through the wireless channel.
 - This is the essence of **multicarrier transmission** and forms the basis of **OFDM**.

7.2 Multicarrier Transmission

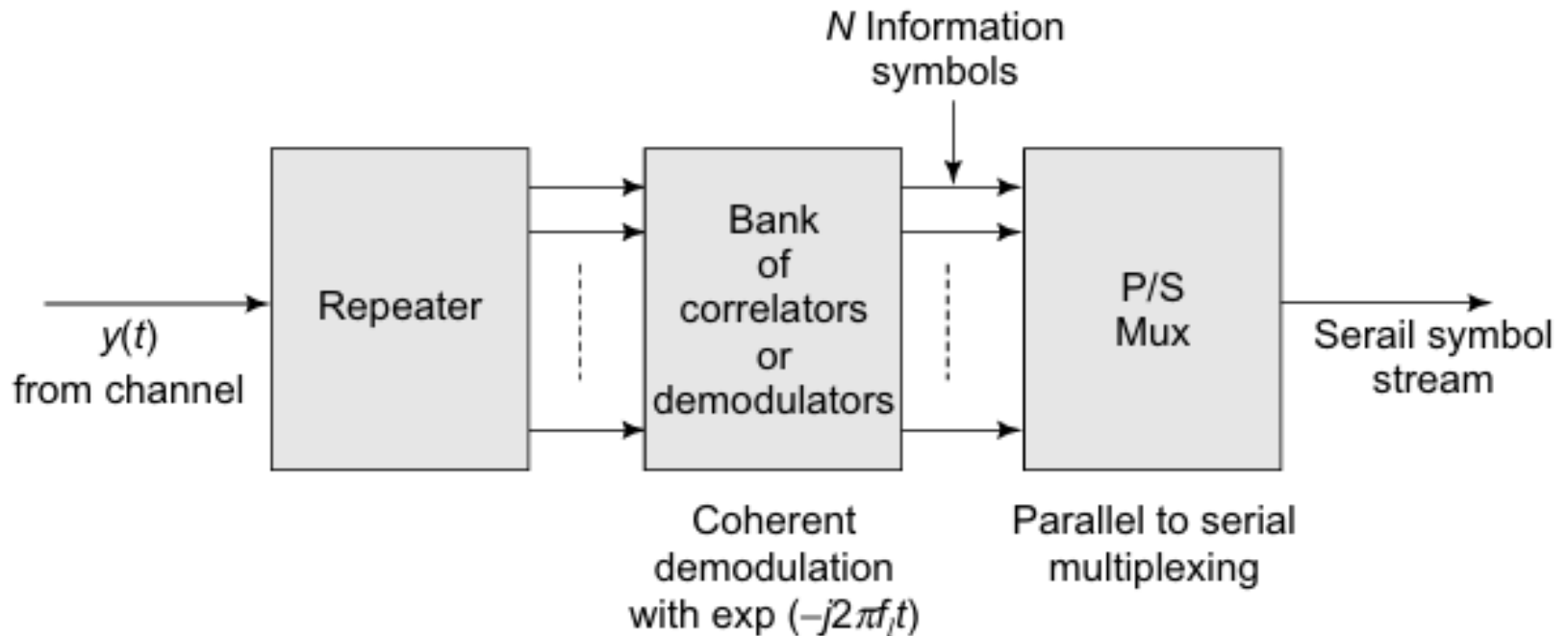


Figure 7.3 *Multi-carrier modulation receiver*



7.2 Multicarrier Transmission

1. Signal Reception and Repeater

- The received composite signal $y(t)$ from the wireless channel first enters the **Repeater** block.
- This ensures signal amplification and conditioning for further processing.

2. Bank of Correlators/Demodulators

- Each subcarrier is separated using a **bank of correlators**.
- Coherent demodulation is performed by multiplying with $e^{-j2\pi f_i t}$, isolating the i^{th} subcarrier.

3. Extraction of Information Symbols

- The demodulators output N parallel information symbols, each corresponding to its original subcarrier.
- This recovers the parallel data streams transmitted at the transmitter side.

4. Parallel-to-Serial Conversion

- Finally, a **Parallel-to-Serial (P/S) Multiplexer** combines the N recovered parallel streams back into a **serial symbol stream**, reconstructing the original transmitted data sequence.

7.2 Multicarrier Transmission

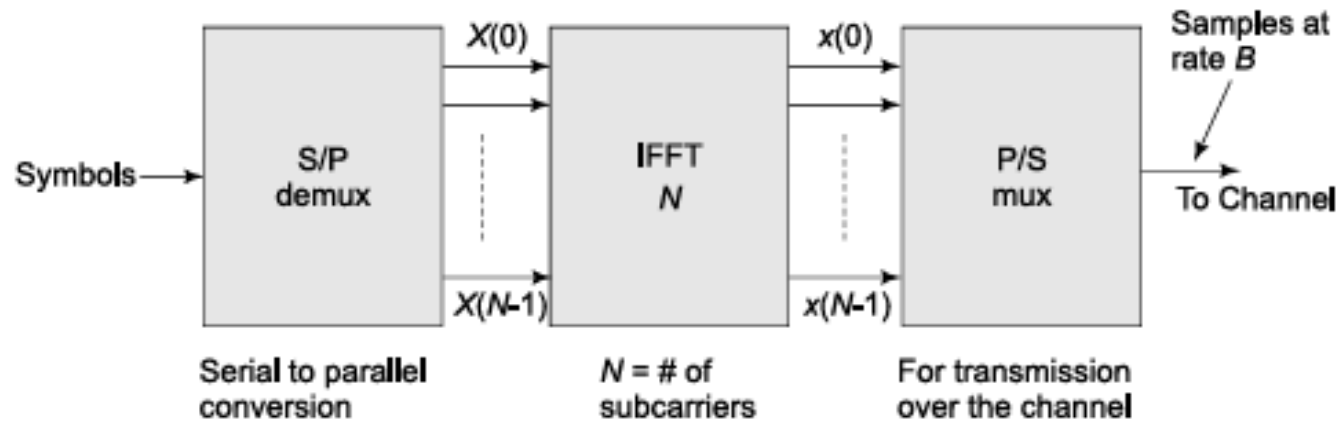


Figure 7.4 *OFDM transmitter schematic with IFFT*

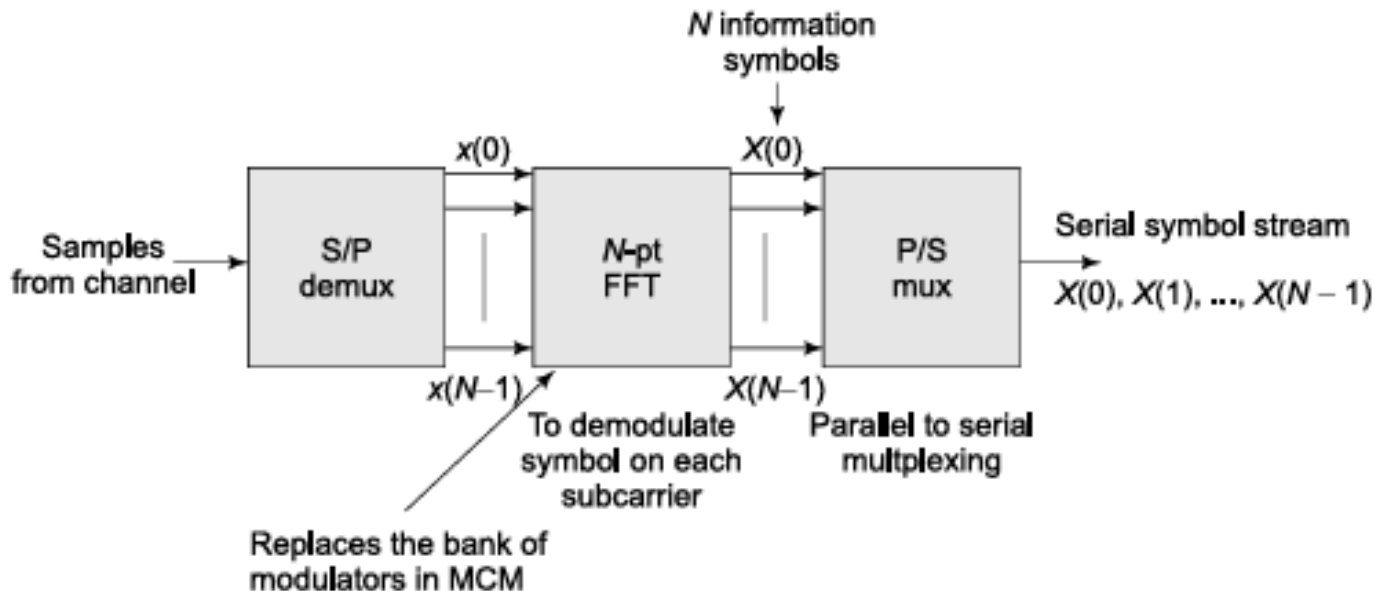


Figure 7.5 *OFDM receiver schematic with FFT*

7.2 Cyclic Prefix

1. Problem of ISI in multipath channels

- In a frequency-selective channel with taps $h(0), h(1), \dots, h(L - 1)$, the received symbol $y(n)$ suffers **Inter-Symbol Interference (ISI)** from the previous $L - 1$ transmitted samples.

2. Introduction of Cyclic Prefix (CP)

- To combat ISI, each OFDM block $[x(0), x(1), \dots, x(N - 1)]$ is **prefixed with the last L_c samples** of the block, i.e., $[x(N - L_c), \dots, x(N - 1), x(0), \dots, x(N - 1)]$.
- This prefix is *cyclic* in nature, hence called the **Cyclic Prefix (CP)**.

3. ISI confined within block

- With CP length $L_c \geq L - 1$, the ISI is restricted to samples from the **same OFDM block** rather than spilling into the next block.
- This eliminates **inter-OFDM symbol interference**.

7.2 Cyclic Prefix

$$\underbrace{\tilde{x}(0), \tilde{x}(1), \dots, \tilde{x}(N-1)}_{\text{Previous block}}, \underbrace{x(N-L_c), x(N-L_c+1), \dots, x(N-1)}_{\text{Cyclic prefix}}, \underbrace{x(0), x(1), \dots, x(N-1)}_{\text{Current block}}.$$

Initial samples, of subject to inter OFDM symbol interference

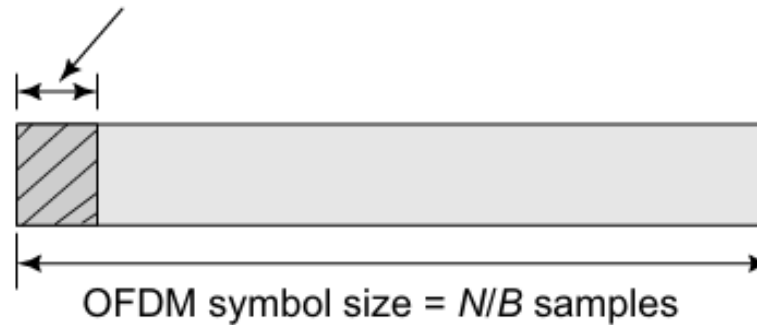


Figure 7.6 *Inter-OFDM symbol interference*

7.2 Cyclic Prefix

Concept of Cyclic Prefix (CP)

- ❖ In multipath fading channels, **linear convolution** of the channel causes **ISI (Inter-Symbol Interference)** between OFDM blocks.
- ❖ Append a **Cyclic Prefix** by copying the **last L_c samples** of an OFDM block and prefixing them to the start.
- ❖ Example:
 - ❖ Original block:
 - ❖ With CP of length 2:
- ❖ ISI remains confined within the same block, and FFT demodulation sees a **circular convolution**, preserving **orthogonality**.

7.2 Cyclic Prefix

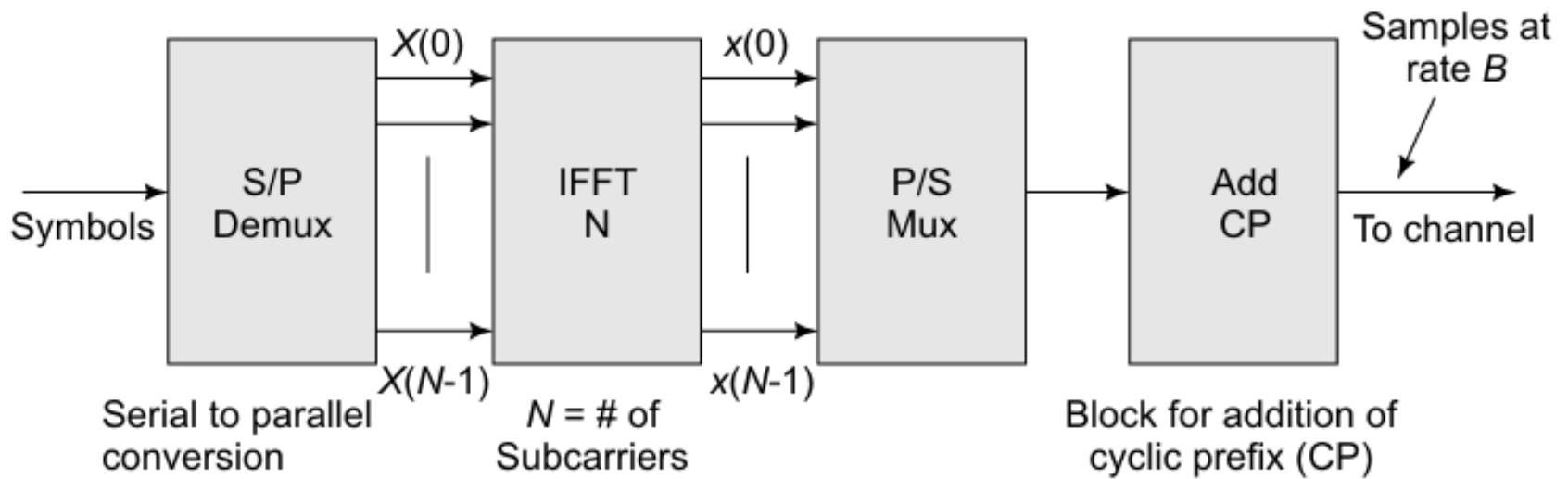


Figure 7.8 *OFDM transmitter schematic with CP*

7.2 Cyclic Prefix

- ❑ Input bitstream is divided into N parallel data streams, each mapped to a subcarrier symbol (e.g., QPSK/QAM).
- ❑ An N -point **Inverse FFT** converts frequency-domain symbols into time-domain OFDM samples $[x(0), x(1), \dots, x(N-1)]$
- ❑ Each subcarrier remains orthogonal due to the FFT basis.
- ❑ The last L_c samples of the IFFT block are copied and inserted at the front:
 $[x(N-L_c), \dots, x(N-1), x(0), \dots, x(N-1)]$
- ❑ This ensures that multipath delay only affects the prefix, not the useful part.
- ❑ Total transmitted block length = $N + L_c$.

7.2 Cyclic Prefix

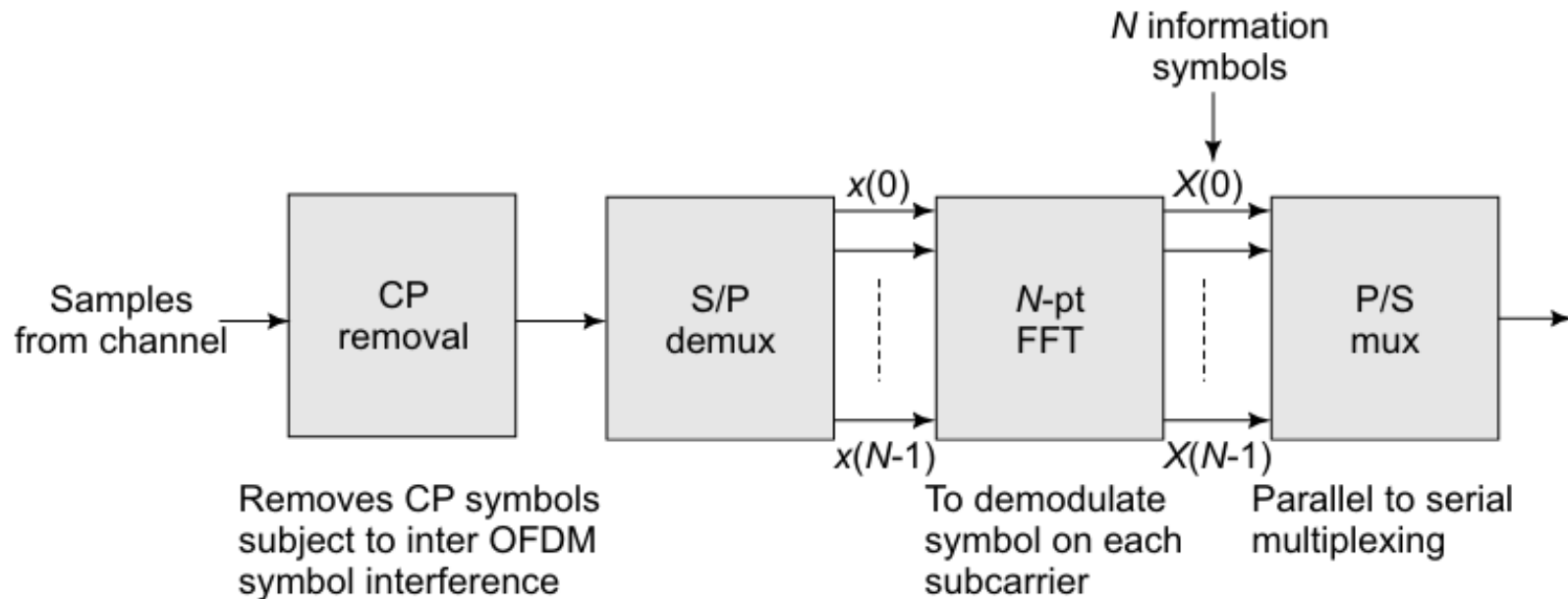


Figure 7.9 *OFDM receiver schematic with CP*

7.2 Cyclic Prefix

- ❑ The channel output includes multipath distortion: $y(n) = h(n) * x(n)$ (linear convolution with channel taps).
- ❑ Receiver discards the first L_c samples, retaining the useful block of $N - L_c$ samples.
- ❑ ISI from previous block is eliminated.
- ❑ An N -point **FFT** is applied: $Y(k) = H(k) \cdot X(k)$,
- ❑ Since each subcarrier is independent, simple **one-tap equalizers** (like Zero Forcing or MMSE) can be applied per subcarrier.
- ❑ The N equalized subcarrier symbols are converted back into a serial data stream, reconstructing the transmitted information.

7.2 Cyclic Prefix – Impact on Data rate

- ❑ The **minimum CP length** must be at least $L-1$, where L is the number of channel taps (delay spread).
- ❑ This ensures that the cyclic prefix covers the maximum multipath delay, preventing inter-symbol interference (ISI).
- ❑ The cyclic prefix samples are just repetitions of the end of the OFDM block.
- ❑ Since they add no new data, they reduce throughput efficiency.

The efficiency loss is given by:

$$\text{Loss} = \frac{L - 1}{N + (L - 1)}$$

where N is the block size.

Larger CP $\Rightarrow$ higher overhead $\Rightarrow$ lower spectral efficiency .

7.2 Cyclic Prefix – Impact on Data rate

As the number of subcarriers N increases:

- The OFDM symbol duration increases ($T = N/B$).
- The relative CP overhead becomes negligible:

$$\lim_{N \rightarrow \infty} \frac{L-1}{N + (L-1)} \rightarrow 0$$

- Thus, throughput loss decreases for larger N .
- Large N means longer OFDM symbol time $\rightarrow$ less ISI, better efficiency.
- But it also increases **decoding delay** since the receiver must wait for all N samples before FFT processing.

7.2 Cyclic Prefix – Impact on Data rate

- CP must satisfy:

$$L_c \times T_s \geq T_d$$

where $T_s = \frac{1}{B}$ is sample time and $T_d = \frac{1}{B_c}$ is channel delay spread.

- For efficient data rate:

$$N \gg L_c$$

or equivalently, subcarrier bandwidth $\frac{B}{N} \ll B_c$, ensuring **flat fading** per subcarrier .

7.3 OFDM EXAMPLE - WiMAX

- Number of subcarriers: $N = 256$
- Subcarrier spacing (given): $B_s = 15.625$ kHz
- Total system bandwidth: $B = N \times B_s = 256 \times 15.625$ kHz = 4 MHz.

Step 1 — OFDM symbol time (without CP)

Symbol time (useful part) is

$$T_{\text{sym}} = \frac{N}{B} = \frac{256}{4 \times 10^6} = 64 \mu\text{s}.$$

Step 2 — Cyclic prefix chosen (WiMAX example)

WiMAX uses CP = 12.5% of symbol time.

$$T_{\text{CP}} = 0.125 \times T_{\text{sym}} = 0.125 \times 64 \mu\text{s} = 8 \mu\text{s}.$$

7.3 OFDM EXAMPLE - WiMAX

Step 3 — CP length in samples

Sample time $T_s = 1/B = 1/(4 \times 10^6) = 0.25 \mu\text{s}$.

Number of CP samples:

$$L_c = \frac{T_{\text{CP}}}{T_s} = \frac{8 \mu\text{s}}{0.25 \mu\text{s}} = 32 \text{ samples.}$$

Total transmitted OFDM block length (samples) = $N + L_c = 256 + 32 = 288$.

Step 4 — Loss in spectral efficiency (overhead)

Loss fraction due to CP:

$$\text{Loss} = \frac{L_c}{N + L_c} = \frac{32}{288} = \frac{8}{72} \approx 0.1111 = 11.11\%.$$

So useful symbol time = $64 \mu\text{s}$, total transmitted time = $72 \mu\text{s}$; overhead = $8 \mu\text{s} \rightarrow 11.1\%$ loss.

7.3 OFDM EXAMPLE - WiMAX

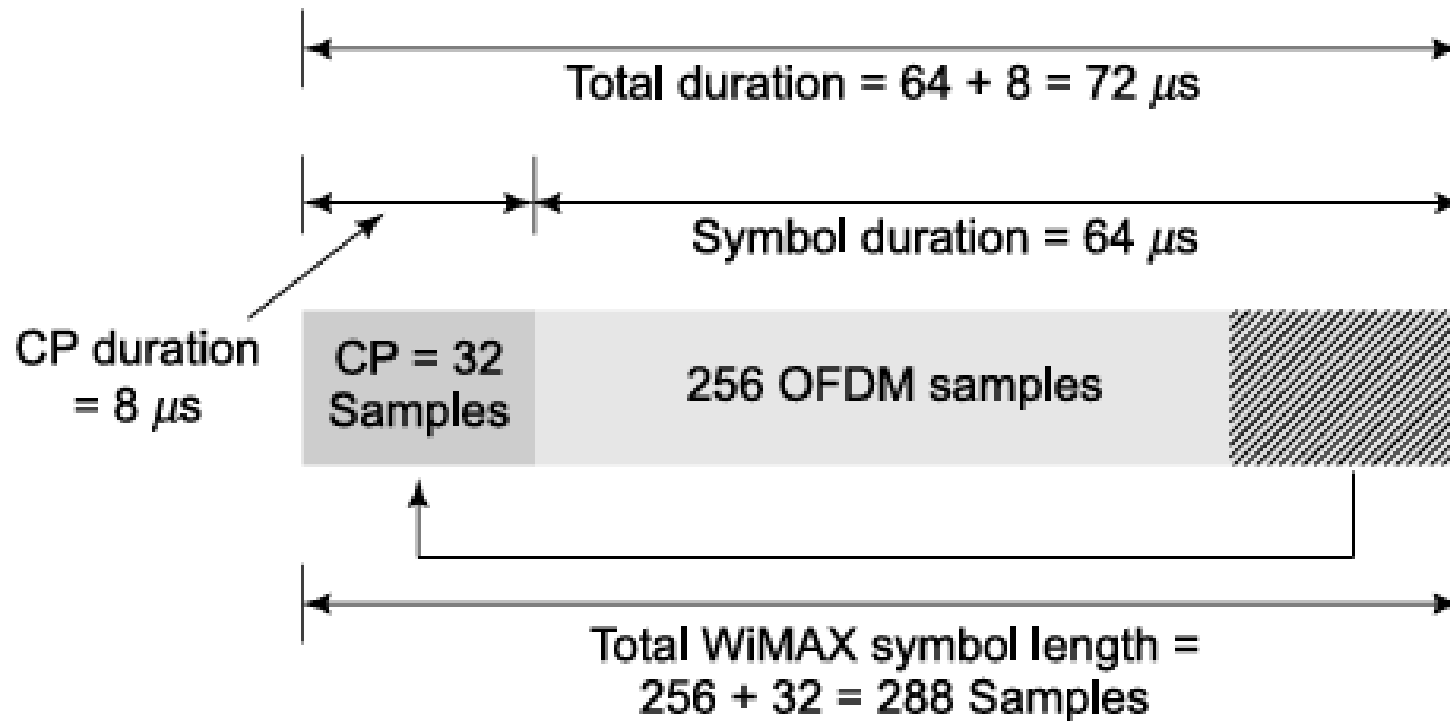


Figure 7.11 *WiMAX OFDM symbol with cyclic prefix*

7.5 MIMO-OFDM

- ❖ **MIMO-OFDM** is a combination of the Multiple-Input Multiple-Output (MIMO) wireless technology with that of OFDM, to further increase the rate in broadband multi-antenna wireless systems.
- ❖ MIMO-OFDM converts a frequency-selective MIMO channel into multiple parallel flat fading MIMO channels.
- ❖ Hence, MIMO-OFDM significantly simplifies baseband receive processing by eliminating the need for a complex MIMO equalizer.

$$\begin{aligned}
 y(n) &= \sum_{l=0}^{L-1} h(l) x(n-l) + w(n), \\
 &= h(0) x(n) + \underbrace{h(1) x(n-1) + \dots + h(L-1) x(n-L+1)}_{\text{ISI from previous symbols}} + w(n)
 \end{aligned}$$

7.5 MIMO-OFDM

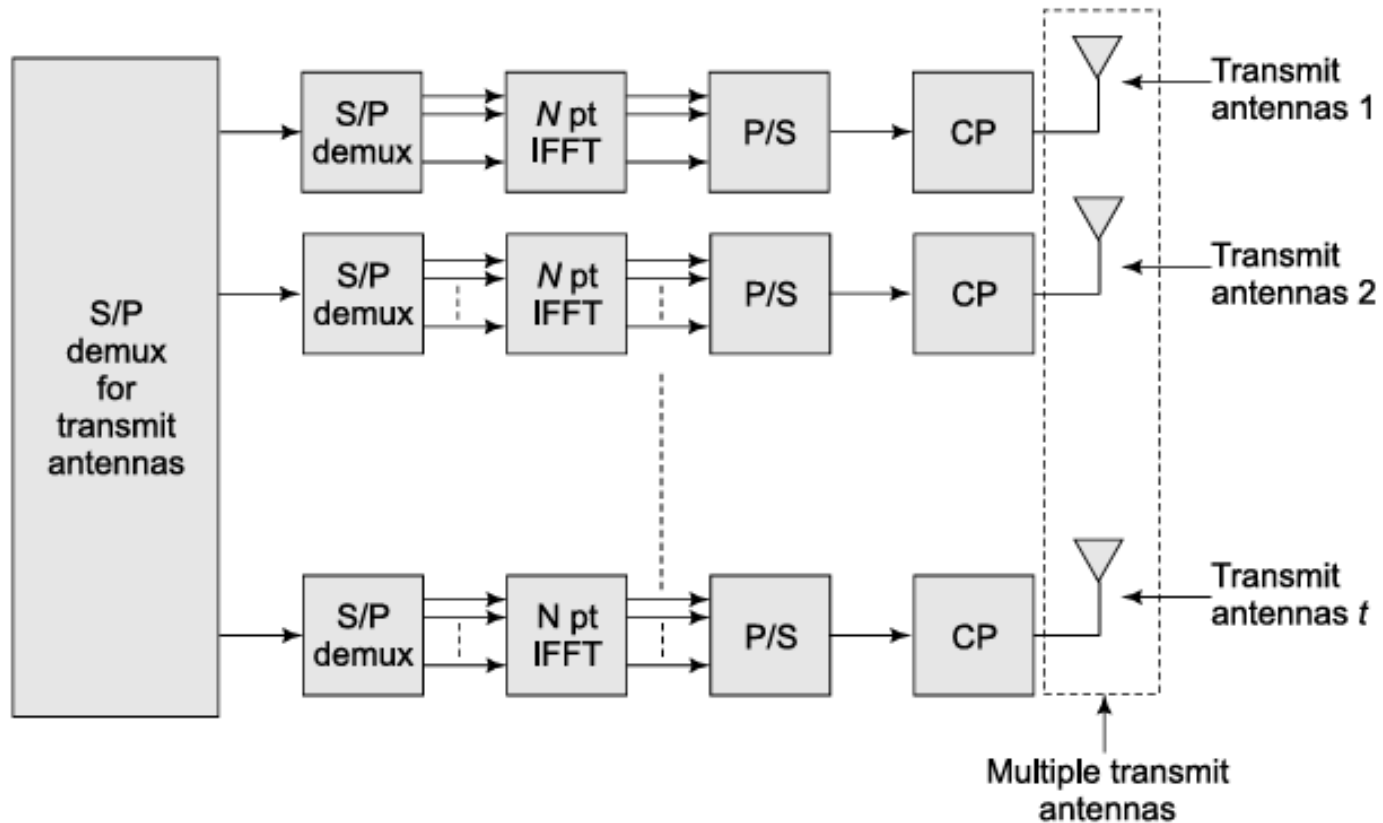


Figure 7.12 *MIMO OFDM transmitter schematic*

7.5 MIMO-OFDM

1. **Input serial bitstream** → converted into parallel streams.
2. **MIMO mapping:** Parallel data symbols are distributed across multiple transmit antennas.
3. **OFDM modulation:**
 - Each antenna applies IFFT to map symbols onto N orthogonal subcarriers.
 - Cyclic Prefix (CP) is added to each OFDM block.
4. **Wireless channel:**
 - Each transmit antenna signal passes through a frequency-selective MIMO channel.
 - Channel matrix $H(f)$ represents path gains per subcarrier.
5. **Received signal per subcarrier (for subcarrier k):**

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k$$

where $\mathbf{H}_k$ is the MIMO channel matrix, $\mathbf{x}_k$ transmitted vector, $\mathbf{y}_k$ received vector.

7.5 MIMO-OFDM

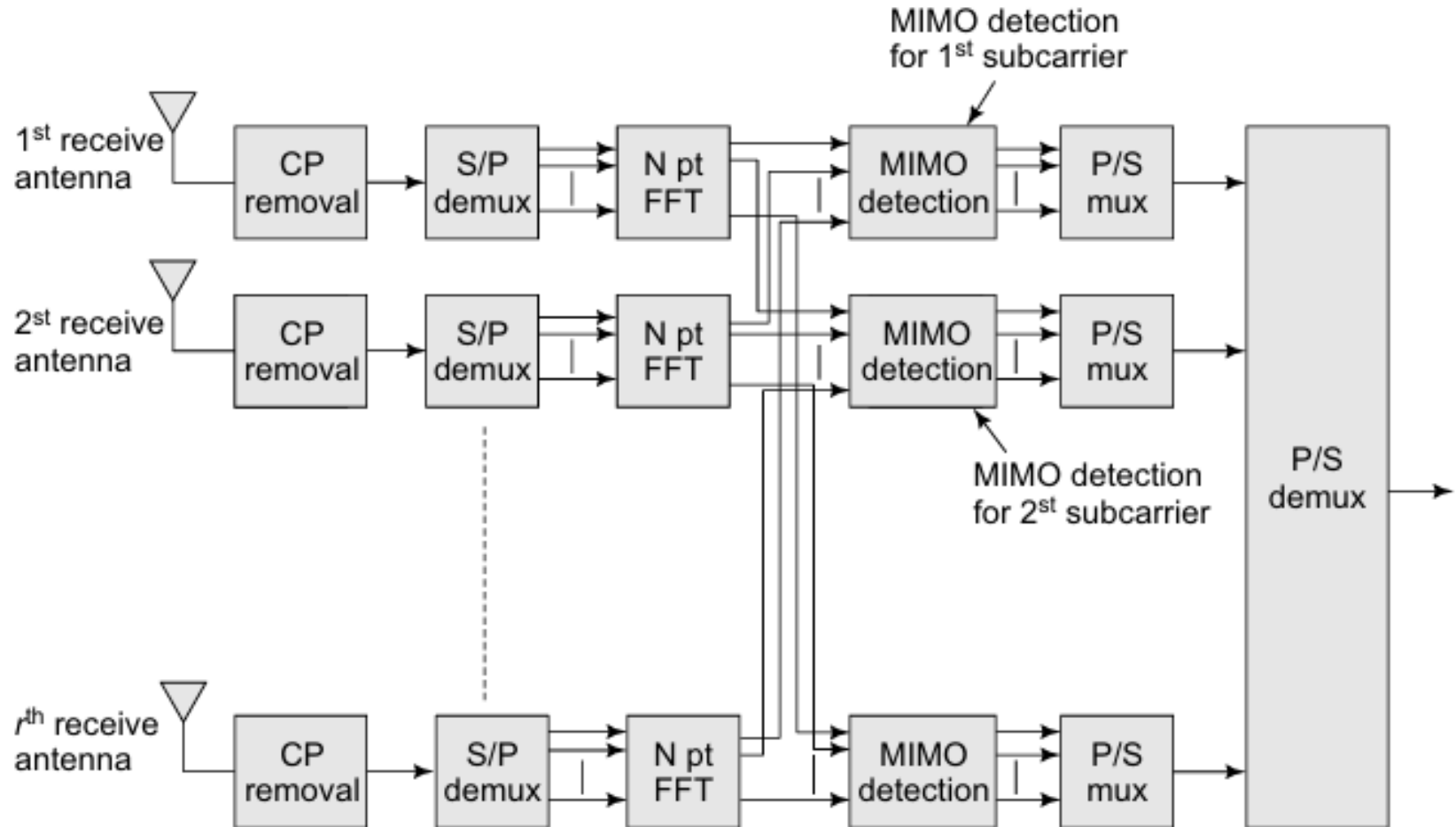


Figure 7.13 *MIMO-OFDM receiver schematic*

7.5 MIMO-OFDM

1. **Cyclic Prefix Removal** → FFT applied to each antenna stream to recover subcarriers.
2. **MIMO detection (per subcarrier):**
 - Zero-Forcing (ZF) or MMSE equalization.
 - Optionally, **Space-Time Coding (STBC)** (e.g., Alamouti) for diversity.
3. **Parallel-to-Serial Conversion** → reconstruct original bitstream.
4. **Advantages of MIMO-OFDM:**
 - High spectral efficiency (supports multi-Gbps rates).
 - Robust against multipath fading.
 - Exploits both **frequency diversity (OFDM)** and **spatial diversity (MIMO)**.
 - Scalable for modern wireless standards.

7.7 OFDM PAPR

- ❖ Peak to Average Peak Ratio is the ratio of the **peak instantaneous power** of an OFDM signal to its **average power**.
- ❖ Critical problem – Need to handled effectively – Limit the distortion at the receiver

$$\text{PAPR} = \frac{\max |x(t)|^2}{E[|x(t)|^2]}$$

$$\text{OFDM PAPR} = \frac{a^2}{a^2/N} = N$$

7.7 OFDM PAPR

- ❖ With N subcarriers, peaks may reach values close to $\sqrt{N}$ times the average.
- ❖ More subcarriers $\rightarrow$ Higher chance of high peaks.
- ❖ Power Amplifiers (PAs) must handle these peaks without distortion.
- ❖ Leads to **back-off operation** $\rightarrow$ less efficiency, more power consumption.
- ❖ Nonlinear distortion causes **Inter-Carrier Interference (ICI)** and **out-of-band radiation**.
- ❖ Degrades Bit-Error Rate (BER) performance.

7.7 OFDM PAPR - IMPACT

- ❖ Receiver **amplifiers operate linearly** only within a limited dynamic range around a bias point. In OFDM, high peaks often exceed this range, pushing the amplifier into **nonlinear operation**.
- ❖ Nonlinear distortion causes loss of subcarrier orthogonality. This introduces **Inter-Carrier Interference (ICI)** and increases the **Bit Error Rate (BER)**.
- ❖ A modified form of OFDM called **SC-FDMA (Single Carrier Frequency Division Multiple Access)** can significantly reduce PAPR

7.8 SC - FDMA

- ❖ **Problem with OFDM: High PAPR (Peak-to-Average Power Ratio)** stresses the transmitter power amplifier.
- ❖ **SC-FDMA (Single-Carrier Frequency Division Multiple Access) -**
A modified form of OFDM that introduces an **M-point FFT before the IFFT**.
- ❖ This effectively retains a **single-carrier nature**, drastically reducing PAPR.
- ❖ PAPR approaches **0 dB**, like single-carrier systems.
- ❖ Used in **LTE uplink** because mobile devices need power-efficient transmission .

7.8 SC - FDMA

- ❖ **Input symbols** → S/P demux → M-point FFT.
- ❖ **Subcarrier mapping:** FFT outputs mapped onto N subcarriers.
- ❖ **N-point IFFT:** Converts mapped symbols back to time-domain.
- ❖ **CP addition + P/S mux:** Generates transmit signal.

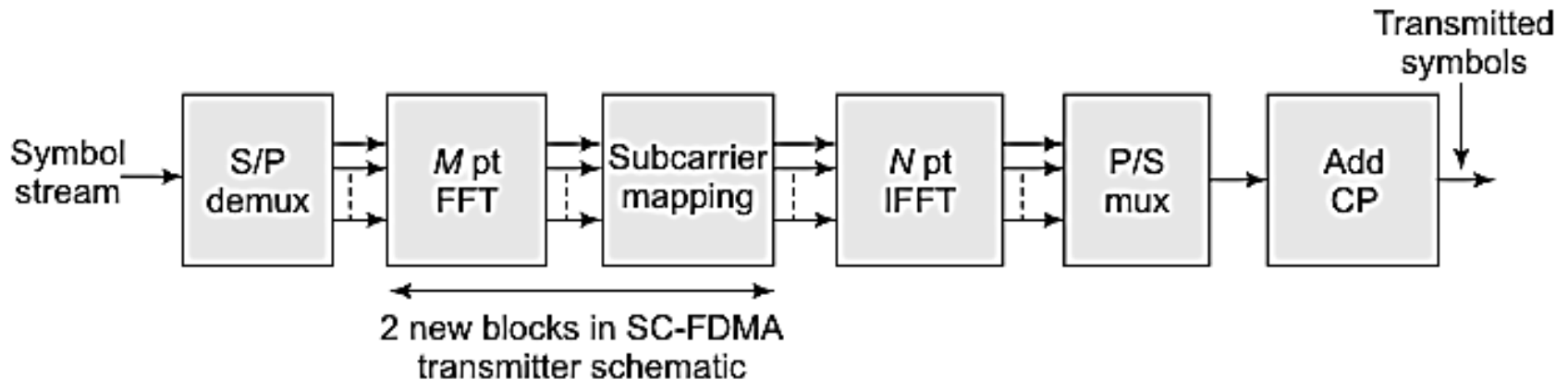


Figure 7.18 SC-FDMA transmitter schematic

7.8 SC - FDMA

- ❖ **CP removal** → N-point FFT.
- ❖ **Equalization**: Compensates for fading channel per subcarrier.
- ❖ **Demapping**: From N subcarriers back to M samples.
- ❖ **M-point IFFT**: Recovers original symbol stream

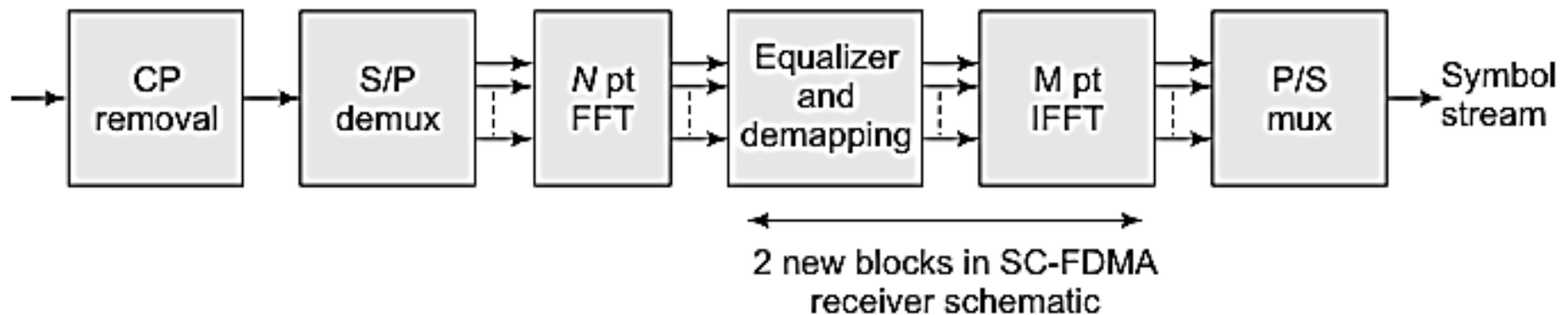


Figure 7.19 *SC-FDMA receiver schematic*

7.8 SC - FDMA

- ❖ Determines how FFT outputs (M samples) are assigned to N subcarriers.
- ❖ **Interleaved FDMA (IFDMA):**
 - ❖ FFT outputs spread evenly across subcarriers with spacing.
 - ❖ Low PAPR, good frequency diversity.
- ❖ **Localized FDMA (LFDMA):**
 - ❖ FFT outputs mapped to a block of consecutive subcarriers.
 - ❖ Simpler implementation, used in **LTE uplink**.

7.8 SC - FDMA

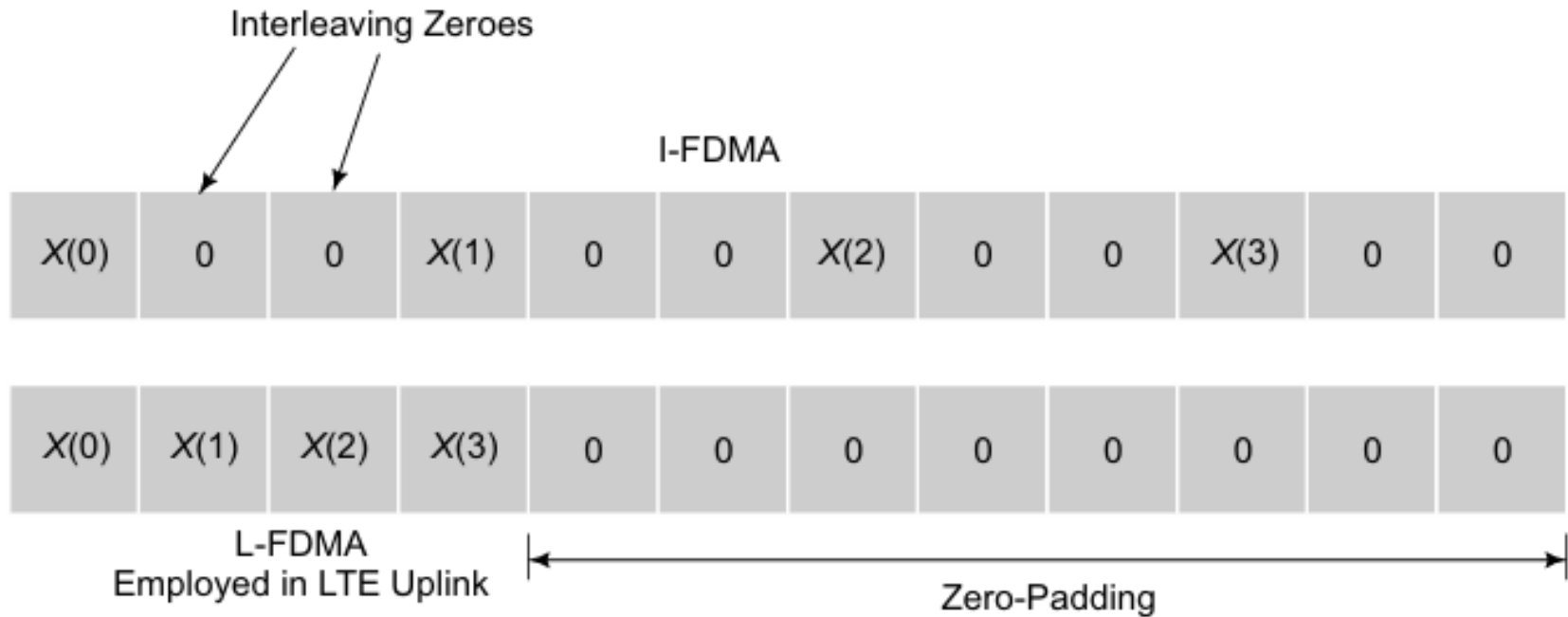


Figure 7.20 SC-FDMA various subcarrier mapping schemes



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THANK YOU.....